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LABORATORY EXERCISES
IN
ELEMENTARY PHYSICS

FRANKLIN H. AYRES

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LABORATORY EXERCISES
IN
ELEMENTARY PHYSICS

A MANUAL FOR STUDENTS IN ACADEMIES AND HIGH SCHOOLS

BY

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PREFACE

THIS little work is, as its name implies, simply a manual of exercises in elementary physics. No claim is made that it embraces all the exercises suitable for work of this grade. It is believed, however, that the exercises as herein presented constitute a sound basis for a strong, rational, and flexible course in elementary laboratory physics.

There has been no attempt in this book to usurp the functions of the teacher. Much of the details of the work are most effective when they come directly from him. The inspiration must also come, in large measure, from the instructor; no book can take his place in this particular.

In handling laboratory work in physics of any grade, the instructor is confronted at all times with the serious problem of providing apparatus. This problem is all the more serious in view of the fact that good work, even in elementary physics, is quite impossible without reasonably good apparatus. Speaking to this very point, Professor Nichols has said: "The list of instruments essential to the carrying on of an effective course in laboratory physics is not a large one. It includes certain standard apparatus, such as the balance, the air-pump, the thermometer, the projecting lantern, the electrical machine, and the galvanometer; together with such accessories as are necessary to use with these instruments. Without such an equipment, no college or school should attempt to give instruction in physics."

The meaning of Professor Nichols's final observation, just quoted, which is confirmed by my own experience, is that effective work in physics is impossible with odds and ends, with mere models to serve as apparatus. On the other hand, too great refinement of method and result should neither be attempted by students of high-school grade nor expected of them. Nevertheless, generally speaking, apparatus which is not accurate within five per cent, at least, should never be put into the hands of such students. Both the possibility and the probability of reasonably accurate results should always be present, not only for the reason that the very nature of the subject and the spirit of science-teaching demand it, but for the sake of the moral effect upon the student as well.

So, in selecting the exercises for this manual, I have endeavored to keep ideal conditions constantly in mind. Wherever possible, however, I have suggested alternative methods and apparatus in order to better adapt the book to laboratories having a limited equipment.

A shop is strongly recommended to all schools, and especially to those of small means. A comparatively small sum spent in fitting up such a shop will reduce the expense of maintaining a physical laboratory by more than one-half and make possible a vastly greater variety of work. These statements are verified by my own experience.

Acknowledgments are due Mr. E. M. Bainter, of the Department of Mathematics of Central High School, for valuable suggestions, and for reading the manuscript of the manual; and to Mr. Benjamin Lubschez, a recent student of this school, for the drawings from which the illustrations were made.

F. H. A.

KANSAS CITY, MO., *July, 1901.*

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LABORATORY EXERCISES IN ELEMENTARY PHYSICS

INTRODUCTORY

THE WORK OF THE LABORATORY

THE average student enters the laboratory with extremely hazy ideas touching the purpose and value of the work he is to do. This uncertainty should not be allowed to continue, since, if it does, the work must signally fail to accomplish its true object. Now, the nature of this object is determined by the relation which exists between the recitation and the operations of the laboratory.

In the recitation and lecture the general principles and applications of the science are developed as thoroughly as possible. Numerous experiments are performed to illustrate and verify these principles. But mark this: these illustrations and verifications rarely do more than to exemplify the qualitative relations which obtain. The verification of the quantitative relations must, in general, be left to the laboratory. So it may be said that the recitation and lecture blaze out the way for a more definite apprehension of the laws of physics by means of individual work in the laboratory

In these verifications the student is called upon (1) to *observe* carefully and accurately; (2) to *describe* with clearness and precision; (3) to *think* keenly and consecutively; and (4) through these, to *formulate exact*, *unambiguous* *statements* of his conclusions. These powers the average uncultivated human mind possesses in some degree. But, without intelligent and careful development, they rarely pass the stage of mediocrity. To develop these powers is therefore the highest and best object of laboratory training. So, in order to get the most out of this training, the student will do well to observe the following suggestions:

At the very beginning he should disabuse his mind of the idea that his laboratory work is to consist of *aimless toying* with apparatus.

Each exercise should be as thoroughly studied as a lesson in mathematics. If the exercise for the day has been well digested, the student will be able to proceed in most cases without assistance.

Limit communication to matters directly pertaining to the experiment being performed.

Very often two operations may be carried on simultaneously.

Cultivate the habit of observing every detail of the experiment, no matter how insignificant any one may seem.

Keep your note-book open and in it neatly record the results of the experiment at the time they are obtained.

Use decimal fractions. Common fractions will not be accepted.

Make it an inflexible rule to record every measurement and determination, and always in such a form that the meaning of the data shall be perfectly clear and self-explanatory.

That "haste makes waste" is nowhere more true or more fatal to good work than in the laboratory. Therefore make all observations with great care and deliberation. Make it a rule to verify each and every determination.

Never prejudice your good judgment by over-anxiety to have a result come out right. As a matter of fact, this attitude is apt to lead to a result far more erroneous than would have been the case had the observer maintained an open mind. The nearer the experimenter can approximate to being an intelligent, discriminating automaton, the more trustworthy will be his results.

Repeat an observation as many times as the time allowed will permit. That two results agree is no evidence in itself that an accurate determination has been made. On the contrary, broadly speaking, results which differ by a reasonable amount are more trustworthy than those which agree very closely.

Never record a result that is "about so and so." Always estimate a fractional quantity in tenths. If it is more than one-half and less than three-fourths, the result is likely to be more correct if it is estimated to be 0.6 or 0.7 than if it is set down as either one-half or three-fourths.

In computation, much time is wasted by numerical mistakes and unnecessary calculations. Watch the decimal point as you would a chronic disturber of the peace. Never retain more than one doubtful figure in any result. For example, in the operation which is indicated below, suppose the canceled figures to be uncertain. Let us compare the quotient obtained when no figures are dropped with that which is obtained when only one uncertain figure is retained.

$$\frac{25.\cancel{4}\cancel{4}\cancel{5} \times 168.\cancel{5}\cancel{2}\cancel{5}}{1465.2\cancel{2}\cancel{5}} = ?$$

$$\begin{array}{r}
168.525 \\
25.445 \\
\hline
842625 \\
674100 \\
674100 \\
842625 \\
337050 \\
\hline
4288.118625
\end{array}
\qquad\qquad
\begin{array}{r|l|l}
\underline{1465.225} & 4288.118625 & \underline{2.926+} \\
 & \underline{2930450} & \\
 & 13576686 & \\
 & \underline{13187025} & \\
 & 3896612 & \\
 & \underline{2930450} & \\
 & 9661625 & \\
 & \underline{8791350} & \\
 & 870275 &
\end{array}$$

Contrast the above laborious calculation with the comparatively easy one which follows:

$$\begin{array}{r} 168.5\\ \underline{25.4}\\ 6740\\ 8425\\ \underline{3370}\\ 4279.9 \end{array} \qquad\qquad \begin{array}{r|l|l} \underline{1465.22} & 4279.90 & \underline{2.920+}\\ & \underline{293044} & \\ & 1349460 & \\ & \underline{1318698} & \\ & 307620 & \\ & \underline{293044} & \\ & 14576 & \end{array}$$

Now, if the most accurate factor entering into the above results is in error by an amount as small as 1 per cent, it follows that the quotient is also in error to this extent, at least. But the above results do not differ by as much as 1 per cent. Therefore nothing has been gained by performing the full computation.

THE NOTE-BOOK

The note-book should be of convenient size. It should have a durable cover, and should contain paper suitable for writing with pen and ink.

In this note-book are to be kept both the original and the expanded notes on all experiments actually performed.

The student for whom a college course is even a remote possibility should preserve these notes, since they are now a requirement for admission into many colleges and universities.

Each report should consist of two parts:

(A) **Original Data and Observations.**—This part of the report should be written up in the laboratory at the time the experiment is performed. The subject-matter of Part A should be treated under the following heads:

(1) OBJECT OF THE EXPERIMENT.—If the experiment involves the proof of a law or principle which is expressible in the form of an equation or a propor-

tion, include the statement of such fact under this head. For example: "The object of this experiment is to prove that the time of one vibration of a pendulum is proportional to the square root of the length of the pendulum. That is, I am required to prove that

$$T : T_1 :: \sqrt{L} : \sqrt{L_1},$$

in which T is the time and L is the length of one pendulum, and T_1 is the time and L_1 is the length of any other pendulum."

(2) DATA AND RESULTS.—Under this head record all numerical data in tabular form, whenever possible. Following this, record such other observations as have a direct bearing upon the main conclusion to be drawn from the experiment.

(3) THE APPARATUS.—Make a neat pencil drawing of the apparatus which is used for the first time. Index the principal parts, and underneath the sketch write the technical name of the apparatus, if it has one.

(B) **Expanded Notes.**—These notes may be written up outside of the laboratory. This part of the report should include the following points:

(1) PROCESSES.*—Under this head state clearly and concisely what you did in performing the experiment. (No report will be accepted in which the imperative mode is used.)

(2) CALCULATIONS AND CONCLUSIONS.—Under this head state the principal conclusion to be derived from the experiment, together with such other inferences as may seem to have an important bearing upon the work in hand, or that which is to

---

* The instructor may prefer to include this part of the report in Part A.

come. In no case will it be sufficient merely to state these conclusions. Whenever possible, they should be worked out from the data obtained in the performance of the experiment, and valid reasons should be given therefor.

QUERIES.—Answer all questions, and give the solution of all problems under this head.

ERRORS.—Note the principal sources of error.

NOTE.—Part A of each report should be written on the left-hand page of the note-book.

VARIATION

Nothing in the laboratory experience of the average high-school student causes more trouble than the handling of variables. Many of the laws of physics are conveniently expressed in the form of a proportion. The proportions which are commonly met in elementary physics involve what are known as dependent variables.

When a quantity in a given problem may have an indefinite number of values it is called a variable. A dependent variable is a quantity so related to another quantity that a given change in the latter produces a direct or inverse variation in the former.

DIRECT VARIATION.—Suppose, for example, that we consider the case of pressure in liquids. Verbally stated, pressure in liquids is proportional to the depth; or, pressure varies directly as the depth. This idea may be expressed in the form:

$$P \propto D.$$

Or, as it is usually written,

$$P : D, \text{ or, } \frac{P}{D}.$$

These are merely different ways of expressing a simple, direct ratio. Now, if we wish to express the relation which exists between two pressures, P and P_1, and the respective depths, D and D_1, at which these pressures obtain, we may do so in the form of a simple, direct proportion, thus:

$$P : D :: P_1 : D_1.$$

$$\text{Or, } P : P_1 :: D : D_1.$$

Remember this: a simple, direct proportion is an equality of simple, direct ratios.

INVERSE VARIATION.—Sometimes a dependent variable is so related to another quantity that a given change in the latter produces an *inverse* change or variation in the former. Consider the case of a body of gas confined under pressure. Now, it is a familiar fact that, as the pressure increases, the volume of the gas decreases. So we say the volume of a body of gas *varies inversely* as the pressure—that is,

$$V \propto \frac{I}{P}.$$

Or, as it is usually written,

$$V : \frac{1}{P}, \text{ or, } \frac{V}{\frac{1}{P}}.$$

These are different ways of expressing a simple, inverse ratio. Reasoning as we did in the preceding case, we may express the relation which exists between two pressures, P and P_1, and the respective volumes, V and V_1, which these pressures produce in the form of a simple, *inverse* proportion—inverse, for the reason that as the pressure increases the volume decreases. That is:

$$P : \frac{1}{V} :: P_1 : \frac{1}{V_1}.$$

$$\text{Or, } \quad P : P_1 :: \frac{1}{V} : \frac{1}{V_1}.$$

$$\text{Whence, } P : P_1 :: V_1 : V.$$

Remember again: a simple, inverse proportion is an equality of simple, inverse ratios.

SOURCES OF ERROR*

The extreme fallibility of human judgment is continually in evidence in the laboratory. The pathway of the student is ever beset with tendencies toward error. Now, errors are *not* mistakes. The former, in some degree, are inevitable. The latter are generally due to carelessness. Errors may be classified as follows:

1. Errors due to the method employed.
2. Errors due to the apparatus.
3. Errors of performance.
4. Errors of observation.

While none of these can be completely eliminated, yet if the student is ever on guard, looking for them, their magnitude may be materially reduced.

Errors due to faulty method can only be remedied by changing the method of the experiment. This, however, rarely lies within the ability of the student. Nor, as a rule, is he able to eliminate errors due to apparatus.

Some correction at least may be applied by the beginner to errors of observation. Suppose a series of determinations of the same quantity to be divided into two groups. Those which are greater than the true value may constitute the + group, and those that are less the − group. Experience has shown that, in a series of observations of the same quantity, the + errors are offset by the − errors. Therefore the average of such a series is likely to be more nearly correct than any one of them. In applying this principle the number of observations should be large.

In many instances it is instructive to compute the per cent of error. It often happens that the average of several determinations of the same quantity seems to indicate a large error when only the difference between it and the accepted value is considered. Suppose, for example, that a certain quantity is known to be 34,000. Suppose, further, that this quantity is determined by experiment to be 35,125. Now, this value of the quantity differs from its true value by 1,125. The largeness of this error-difference seems, at first sight, to make the determination unworthy of acceptance. But if this error-difference is reduced to per cent of error by the simple equation,

---

* For a comprehensive discussion of this subject the student is referred to Nichols's Laboratory Manual, vol. i.

$$\text{Per cent of error} = \frac{\text{Error-difference}}{\text{True value}}.$$

We discover that our result is in error by 3.3 per cent—not such a large error, after all, when we remember that the apparatus usually available for high-school work is not, generally speaking, accurate within 5 per cent.

TRIGONOMETRICAL RELATIONS

Two simple trigonometrical relations, or functions, have a limited use in elementary physics. They are the sine and the tangent of an angle.

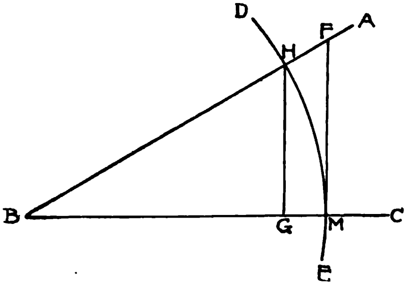


FIG. 1.

Let *ABC* be an angle whose sine and tangent are to be represented.

With *B* as a center, describe the arc *DE*, cutting *AB* at *H* and *BC* at *M*. From *H* let fall *GH* perpendicular to *BC* at *G*. Draw a tangent to *DE* at *M*, intersecting *AB* at *F*.

Now, the sine of an angle is the ratio of the opposite side to the hypotenuse.

Whence, $\sin \angle B = \frac{GH}{BH}$.

Also, the tangent of an angle is the ratio of the opposite side to the adjacent side.

Whence, $\tan \angle B = \frac{FM}{BM}$.

A table of sines and tangents is given in the Appendix.

CHAPTER I

INTRODUCTORY EXERCISES IN MEASUREMENTS

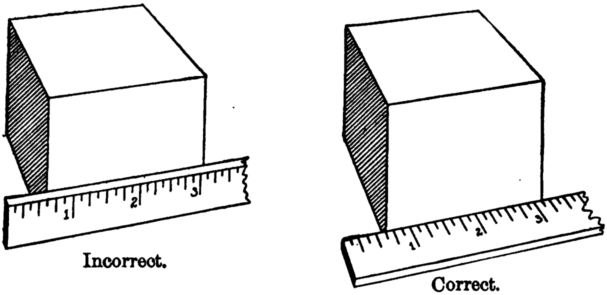
EXERCISE 1

The Common Rule

Apparatus.—A metric rule. A rectangular block of wood.

Directions.—(1) Observe that the rule you are using is a meter-stick, and that it is divided into decimeters, centimeters, and millimeters.

(2) Measure the length, breadth, and thickness of the rectangular block. Make several determinations of each dimension. In measuring the length of any object, place the graduated edge of the rule against the object (see Fig 2). Never use the first and last divisions of the scale; they are liable to be inaccurate. (Why?)



Incorrect.

Correct.

Fig. 2.

(3) Measure the length, width, and height of the table at which you work. Make several determinations of each dimension.

(4) Measure both diagonals of the table-top. Make several determinations, as before. (The table-top is assumed to be rectangular.)

Record all data in the form which follows. Omit no determination.

OBJECT.	Length.	Breadth.	Thickness.	Measured diagonal.	Computed diagonal.
Block.	....	....	....	⋱	⋱
Mean.					
Table.	....	....	....	⋱	⋱
Mean.					
Table-top.	....	....	⋱		
Mean.					

EXERCISE 2

Rule and Caliper Measurements of a Sphere and a Hollow Cylinder

APPARATUS.—A metric rule. A wooden straight-edge and two squarely cut blocks. A small hollow cylinder. A sphere. Outside and inside calipers.

DIRECTIONS.

PART A.—**To take measurements of the sphere and the hollow cylinder with straight-edge and blocks.**

(1) Place the straight-edge in a convenient position on the table. Place the blocks firmly against the straight-edge, with the sphere held tightly between the blocks, as shown in the figure. Observing the caution given in (2) of Exercise 1, measure the distance between the blocks.

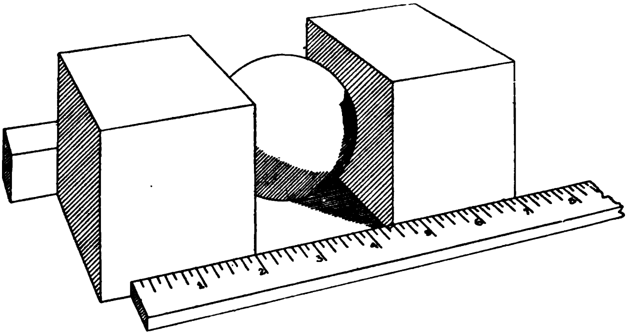


FIG. 3.

(2) Measure the outside and inside diameters of the hollow cylinder in the same manner.

(3) Measure the length of the cylinder in the same way also.

PART B.—**To take measurements of the sphere and the hollow cylinder with the calipers.**

DIRECTIONS.—(1) Open the jaws of the calipers to such an extent that, when the

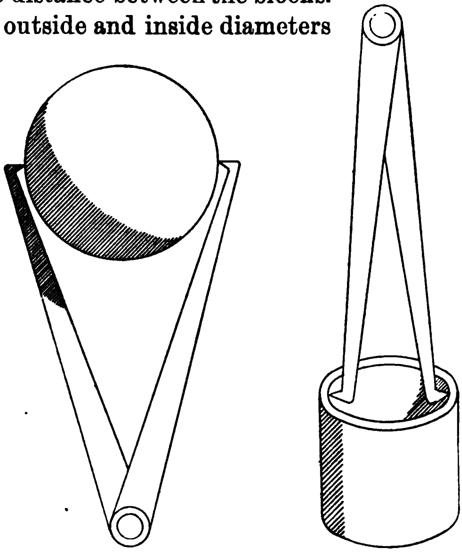


FIG. 4.

points of the jaws of the calipers are placed ***diametrically*** with respect to the sphere, they will just touch the sphere. Without disturbing the calipers, measure the amount of opening between the points of the jaws with the metric rule. ***Estimate the fractional division in tenths.***

(2) Measure the external diameter of the cylinder in the same manner.

(3) Cross the jaws of the calipers to such an extent that when they are placed diametrically within the cylinder they will just fit. Remove the calipers and measure the amount of opening between the jaws.

Record data in the following form:

Object.		Diam. by blocks.	Diam. by calipers.	Vol. by blocks.	Vol. by calipers.
		....	....		
Sphere.		....	....	....	....
		....	....		
Mean.					
		Outside diam.	Inside diam.	Length.	Vol. of shell.
Hollow cylinder	By blocks.	....	....	....	
		....	....	....	....
		....	....	....	
	Mean.				
	By calipers.	....	....		
		....	....	....	....
		....	....		
	Mean.				

Conclusions.—Draw a conclusion touching the comparative accuracy of the two methods used in this exercise. To illustrate: Suppose the outside diameter of the hollow cylinder is found to be 3.4 cm. as measured with the calipers, and 3.6 cm. as measured with the blocks.

Which result, in your opinion, is more nearly correct? Why?

NOTE.—Do not make the calculations called for in columns (4) and (5) of the tabular form until all determinations have been made.

EXERCISE 3

The Diagonal Scale *

APPARATUS.—A diagonal scale. Strips of brass, on each of which are ruled two small crosses. Two finely divided scales, one in metric and the other in English units.

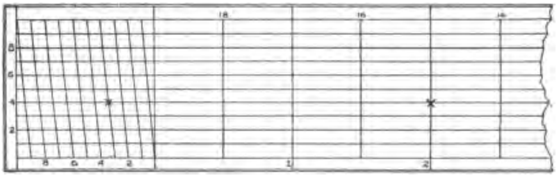


FIG. 5.

PRELIMINARY STATEMENT.—It is required to measure, first with dividers and ordinary scales, then with dividers and diagonal scale, the distance between the centers of the two small crosses on each strip of brass, and to compare the accuracy of the two methods.

PART A.—**To take measurements with dividers and ordinary scales.**

DIRECTIONS.—(1) Apply the dividers to the strip of brass so that the points of the dividers shall coincide, respectively, with the centers of the two crosses; then apply them to each of the common scales. Make as close readings as possible. Estimate all fractional divisions in tenths.

PART B.—**To take measurements with dividers and diagonal scale.**

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* Stewart and Gee, vol. i, pp. 4, 5, 6.

Preliminary Discussion.—Both the method of using and the theory of the diagonal scale will be better understood after a consideration of the following points:

(1) Observe that the diagonally ruled space is an *auxiliary* scale division.

(2) Observe that the auxiliary scale division is divided, each way, into ten equal parts. Observe, further, that the first diagonal line—and the first horizontal line, as well—is numbered zero.

(3) The principle of the diagonal scale may be made perfectly clear by the following questions:

(*a*) How far is it from cross line 1 to diagonal line 1, measured on horizontal line zero? To diagonal line 5, measured on the same horizontal?

(*b*) How far is it from cross line 3 to diagonal line 1, measured on horizontal line 2? To diagonal line 7, measured on horizontal line 9?

Directions.—(1) Not having disturbed the dividers, apply them to the diagonal scale so that one point shall fall on one of the principal cross lines of the scale and the other one shall fall within the diagonally ruled space—both points being, of course, on the same horizontal.

(2) Now move the dividers directly across the scale to such a position that, with both points on the same horizontal line and one of them still on the principal cross line referred to in (1), the other point will fall at the intersection of a diagonal line with a horizontal line. Record readings in the following form:

Strip No........

Principal scale reading......................
Diagonal line reading.......................
Horizontal line reading.....................

Note.—Do not stop at this point to make any calculations. *Never* do so during the progress of an experiment, unless it be absolutely necessary. Cultivate the habit of recording data with such fullness and clearness that any calculation pertaining to the experiment may be made at any time.

(3) In the same way measure the distance between the crosses on each of the other strips. Record all readings in parallel columns, in the form suggested above.

Record results in the following form:

Measurement.	Distance between crosses.		
	Strip No. ..	Strip No. ..	Strip No. ..
English scale.			
Metric scale.			
Diagonal scale.			

Calculations and Results.—Suppose, for example, that the right-hand point of the dividers falls at principal cross line 2 and the left-hand one between diagonal lines 3 and 4—both points being on horizontal line zero, as directed in (1), above. The length under measurement is evidently between 2.3 and 2.4 units of the scale employed—that is, the length is 2.3 +. The second decimal place remains to be determined.

To find the second decimal place, proceed as follows: Move the dividers across the scale until the left-hand point falls at the intersection of a diagonal with a horizontal line. Suppose, when this is done, that the left-hand point falls at the intersection of diagonal 3 with horizontal line 4. The second decimal place sought is 4, and the total distance between the crosses in question is 2.34 half-inches, inches, centimeters, or whatever may happen to be the principal unit of the scale used.

Prove, by geometry, that the distance given in the foregoing example is 2.34.

Make a simple rule for the rapid use of a diagonal scale.

Query.—To how many decimal places do you think a determination by a diagonal scale may be correct?

EXERCISE 4

The Micrometer Caliper*

Apparatus.—A micrometer caliper. Bicycle balls. A thin sheet of metal. A fiber of silk. A bit of fine wire, or a hair.

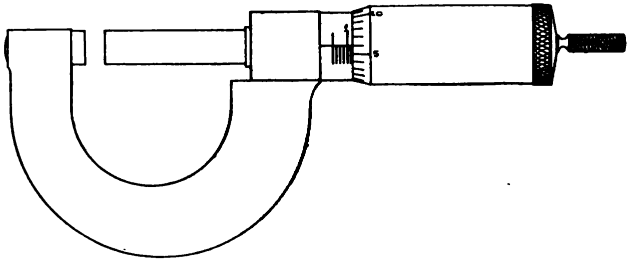

Fig. 6.

Preliminary Discussion.—Whenever it is desired to measure comparatively small lengths with great accuracy, instruments which embody the principle of the screw are employed. An instrument of this class is the micrometer caliper.

Directions.

Part A.—**To measure the diameter of a bicycle ball.**

(1) Place the ball centrally between the jaws of the caliper and turn the screw until the jaws gently pinch the ball. Turn the screw *with a light touch.* Read the number of whole divisions indicated by the *linear* scale. Next, read the number of divisions indicated by the *circular* scale, reading upward toward the horizontal line of the linear scale. Estimate the fractional divisions in tenths.

Part B.—**To measure the thickness of a thin sheet and the diameter of a fine fiber.**

(1) Proceed precisely as directed in (1), above.

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* Stewart and Gee, vol. i, p. 14.

Record all readings in the form which follows:

Body.	Linear scale reading.	Circular scale reading.
Bicycle ball.	....	....
Mean.		
Thin sheet.	....	....
Mean.		
Fine fiber.	....	....
Mean.		

Calculations.—Suppose the "pitch" of the micrometer to be 40—i. e., one revolution of the screw advances it $\frac{1}{40}$ of an inch, or one division of the linear scale. Suppose, further, that the beveled edge of the barrel is divided into 25 equal parts, to constitute the circular scale. Evidently, if the screw be revolved through one division of the circular scale—i. e., through $\frac{1}{25}$ of one complete revolution—the screw will be advanced $\frac{1}{25}$ of $\frac{1}{40}$ of an inch. Wherefore, each division of the circular scale measures $\frac{1}{1000}$ of an inch.

Example.—In measuring the diameter of a certain bicycle ball, suppose the linear scale reading to be 4 and the circular scale reading to be 3.5. Then D, the diameter of the ball in inches, is given by the equation,

$$D = \frac{\text{Linear Reading}}{40} + \frac{\text{Circular Reading}}{1000}.$$

$$\text{Whence, } D = \frac{4}{40} + \frac{35}{1000} = 0.135 \text{ inch.}$$

Results.—Compute the diameter or thickness of each object used. Include these results in your report. Put them in tabular form.

Query.—**Measured with a micrometer caliper, the diameter of a certain fiber is found to be 0.00227 inch. To how many decimal places may this result be correct?**

EXERCISE 5

The Vernier Rule*

Apparatus.—A fixed scale, divided into half-inches, is provided with an auxiliary movable scale which slides over or at the side of the fixed scale. A length of N–1 scale divisions on the vernier is divided into N equal parts. For example, the vernier may be 19 half-inches in length and divided into 20 equal parts; or 9 half-inches in length and divided into 10 equal parts. Sometimes a length of N scale divisions on the vernier is divided into N–1 equal parts. The principle of both methods is the same.

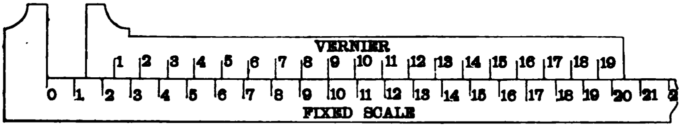

Fig. 7.

Part A.—**To take measurements with a simple vernier.**

Directions.—(1) Place a rod, say one which is between 0.5 inch and 1 inch long, between the zero end-piece of the rule and the zero end of the vernier. Move the vernier up so as to hold the rod firmly.

(2) Suppose the fixed scale to be divided into half-inches. Evidently the length of the rod is equal to the distance between the zero point on the fixed scale and the zero point on the vernier. This distance is equal to one division, or half-inch, of the fixed scale, plus a part of the

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* Hastings and Beach, p. 10; Loudon and McLennan, p. 6; Stewart and Gee, vol. i, p. 7.

next division. The length of this part is obtained thus: Find the division mark of the vernier which most nearly coincides with a division mark of the fixed scale. Suppose it to be the ninth. Then the length of the part in question is $\frac{9}{40}$ of the length of one scale division of the fixed scale, or 0.225 inch.

Whence, the length of the entire rod is 0.725 inch.

(3) Measure the diameter of a bicycle ball with the vernier rule.

(4) Measure the length and thickness of various small bodies in order to become perfectly familiar with the vernier rule.

(5) Measure the diameter and thickness of the objects used as directed in (3) and (4) with a micrometer caliper.

Record all data in a neat tabular form.

RESULTS.—Prove that, in the example given above, the length of the rod is 0.725 inch.

QUERIES.—For the sake of accuracy, should the divisions of the fixed scale be few or many? Name the principal advantages of having a finely divided fixed scale.

PART B.—**To take measurements with more elaborate and accurate verniers.***

APPARATUS.—A vernier caliper. A barometer provided with a vernier.

PRELIMINARY DISCUSSION.—It should be remembered that the vernier reading is always a fraction of the smallest unit of the fixed scale. For example, each inch of the fixed scale of most vernier calipers of American manufacture is divided into fifty equal parts. So the smallest unit of the fixed scale of these instruments is $\frac{1}{50}$ inch. Generally speaking, twenty divisions of the vernier of these instruments correspond to nineteen of the fixed scale. It follows that if the first division mark of the vernier coincides with a division mark of the fixed scale, the length under measurement is $\frac{1}{20}$ of $\frac{1}{50}$ of an inch, or $\frac{1}{1000}$ inch. Such calipers, therefore, measure to thousandths of an inch.

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* For a thorough discussion of the principle of the vernier, see the references cited in Part A.

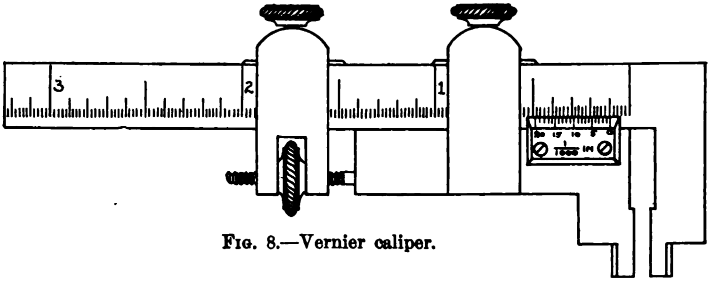

Fig. 8.—Vernier caliper.

Barometer verniers are usually divided as follows: English fixed scale, 20 divisions to the inch, 24 divisions of the fixed scale corresponding to 25 divisions of the vernier scale; metric fixed scale, 10 divisions to the centimeter, 9 divisions of the fixed scale corresponding to 10 divisions of the vernier scale.

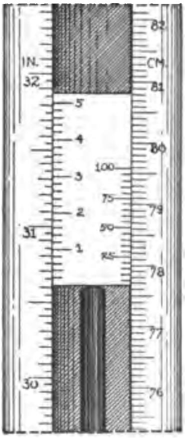

Fig. 9.—Barometer vernier.

Directions.—(1) Record the denomination of the smallest unit of the fixed scale of the instruments furnished you, and the corresponding scale numbers of the fixed and vernier scales of each.

(2) Measure the diameter of a bicycle ball with the vernier calipers. Make several determinations. (See Part A.)

(3) Measure the diameter of the same bicycle ball with a micrometer caliper. Make several determinations.

(4) Take several very careful readings of the barometer, using the vernier.

Record all data in a neat form.

RESULTS.—Show that the vernier referred to in Preliminary Discussion measures to thousandths of an inch.

Determine to what fraction of an inch, and of a millimeter, the barometer vernier mentioned in Preliminary Discussion is capable of measuring.

Compare the vernier calipers with the micrometer caliper with respect to accuracy. Give reasons for your conclusions.

EXERCISE 6

The Spherometer;* Thickness of a Thin Plate

APPARATUS.—A spherometer. A sheet of glass having a true surface or a steel surface plate. A thin plate whose thickness is to be determined.

DIRECTIONS.—(1) Record the denomination of the smallest unit of the linear scale. (The student will observe that the spherometer is merely a modified form of the micrometer screw.) Record the number of divisions composing the circular scale. Record the "pitch" of the screw.

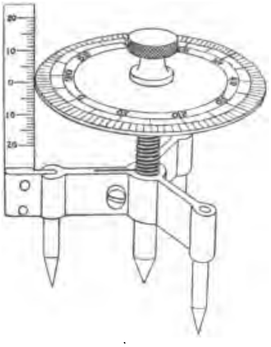

FIG. 10.

(2) Place the instrument on the surface plate and turn the screw until it touches the plate. Be certain that the screw is in contact with the plate and that there is no tendency of the spherometer to rock on its central leg.

(3) Now take the reading of both the linear and circular scales. Take at least four other readings with the instrument at different positions

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* Nichols's Laboratory Manual, vol. i, p. 26. Loudon and McLennan, p. 10.

on the surface plate. From these readings the average *zero* reading is to be computed.

(4) Raise the screw until the thin plate will slip under it. Bring the screw into contact with the plate as directed in (2). Take at least five readings of both scales, with the spherometer at widely different positions on the plate. Call these the *final* readings.

Record data in the following form :

Spherometer No. ....
Linear scale divided into ....
Circular scale of .... divisions.
Pitch of screw ....

TRIAL.	ZERO READINGS.		FINAL READINGS.	
	Linear.	Circular.	Linear.	Circular.
(1) (2) (3) (4) (5) *Average....*				

CALCULATIONS.—From the first set of readings compute the average zero reading of the screw. From the second set compute the average final reading. Now subtract the average zero reading from the average final reading. Suppose, for example, that this difference is as follows : Linear scale difference, 3 ; circular scale difference, 43 divisions of a scale of 100 parts.* This means that the screw made 3.43 revolutions in moving from the average zero position to the average final position. Suppose the pitch of the screw to be 25. Then the distance through which the screw moved—which is to say the thickness of the object—is equal to 3.43 × $\frac{1}{25}$ inch, or 0.137 inch.

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* In determining the circular scale difference, a mistake may be made on account of the fact that the circular scale may read in the direction in which it is revolved. For example, a final reading of the circular scale may be 26 divisions. The *true* reading in this case is found by subtracting 26 from the total number of divisions in the circular scale.

Compute the thickness of the plate used in the experiment.

QUERY.—Of how many decimal places may we be certain in using this instrument?

EXERCISE 7

The Spherometer; Radius of a Spherical Surface

APPARATUS.—A spherometer. A surface plate. A pair of dividers. A diagonal scale. A convex lens.

PRELIMINARY DISCUSSION.—It is required to compute, by means of the equation which follows, the radius of a given spherical surface:

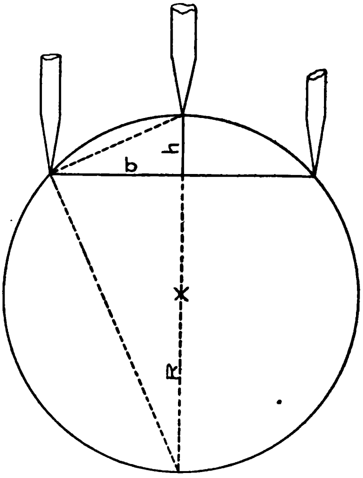

FIG. 11.

If R is the radius of the sphere of which the lens to be used in this exercise is a segment, b is the radius of the base of the segment, and h is the height of the segment, then, by geometry,

$$R = \frac{b^2 + h^2}{2h}.$$

DIRECTIONS. — (1) Determine the average zero reading as directed in the preceding exercise. (This means to take several readings, but *not* to compute their average during the progress of the experiment.)

(2) Turn the spherometer upward until the instrument

will stand firmly when placed on the convex surface of the lens. Be certain that there is not the slightest unsteadiness of the spherometer, as is sure to be the case if the screw has been lowered too far. Take both readings. In the same manner take at least four other sets of readings at widely different positions on the surface of the lens.

(3) Having put a sheet of paper on the surface plate, carefully place the spherometer on the paper so that the legs may make but one set of impressions. Turn the screw downward until it also makes a small dot on the paper. Now, remove the spherometer and measure the distance of the central dot from each of the others with dividers and diagonal scale. Call the *mean* of the three measurements, b.

Record data in a form similar to the one used in the preceding exercise.

CALCULATIONS.—From data obtained as directed in (1) and (2), compute h, the height of the segment. Substitute in the equation given above and solve for R.

Show, by geometry, how the equation, $R = \frac{b^2 + h^2}{2h}$, is derived. Include this demonstration in your report.

EXERCISE 8

The Beam Balance.* Weighing by the Method of Equal Swings

APPARATUS.—A beam balance. A set of weights.

PRELIMINARY DISCUSSION.—Before attempting any work with a good balance, the student should become familiar with the theory, construction, and care of the instrument. (Consult the references cited below.) Furthermore, *no* student should be permitted to use a sensitive balance unless he be willing to conform his weighing practice to the rules which follow:

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* Stewart and Gee, vol. i, pp. 66, 67. Whiting, pp. 27–37. Deschanel, pp. 34–40.

(1) See that the balance is level and that all parts of it are free from dust.

(2) See that the pointer swings through equal arcs on both sides of the center of the scale. If the pointer comes to rest very far from the center of the scale, report such fact to the instructor.

(3) *Always* arrest the beam when the pointer is at, or near, the central position, and do so gently.

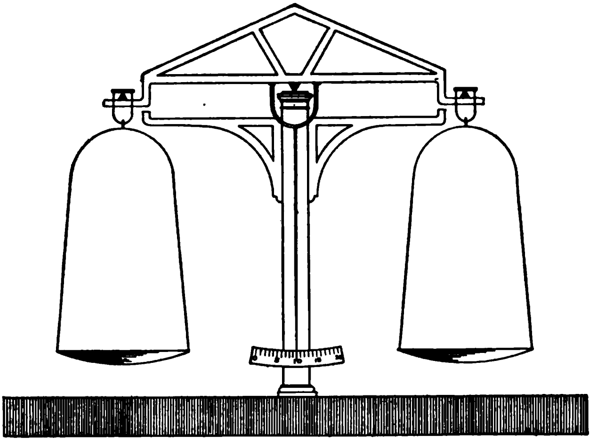

FIG. 12.—Simple form of beam balance.

(4) *Always* arrest the beam whenever weights are to be added or removed.

(5) The position of the observer should be such that the line of sight shall be perpendicular to the plane of the scale of the balance at the middle point of the scale.

(6) Sometimes the equilibrium is so nearly perfect that the balance fails to vibrate. To start vibration, throw a puff of air against one of the pans by a quick movement of the hand.

(7) *Handle all weights with pincers.*

(8) Never weigh bodies while they are hot.

(9) Substances which might injure the pans should be weighed in suitable containers.

(10) Final weighings *must* be made with the case of the balance closed.

(11) At the conclusion of each weighing, arrest the beam, remove the rider from the beam, and place the weights in the box, *in their proper places.*

(12) Keep the weights in the box when they are not in use.

DIRECTIONS.—(1) Place the object to be weighed in the left-hand pan and the weights in the right-hand pan, unless otherwise directed.

(2) Place weights in the proper pan, *in order.* This means to begin with the largest weight which will not overturn the balance, and then to add weights, in the order of their magnitude, until the pointer vibrates as far to one side of the center of the scale as it does to the other. In case the addition of a weight overturns the balance, remove that denomination and try the next smaller. Whenever the smallest weight added does *not* produce perfect equilibrium—which, by the way, is the usual case—a final adjustment may be effected by means of the rider, if the instrument has one.

(3) The rider is a small piece of metal which may be located at any point on the beam. The beam is graduated into such a number of parts that, when the rider is placed at a given scale mark, the number of the mark indicates the *smallest weight* which the balance will *directly* determine. For example, suppose the arms of a certain balance to be divided into ten equal parts. If the balance is sensitive enough to weigh directly to milligrams, the rider should weigh 10 mg. The reason for this is simple enough; for, if 1 mg. be placed in the pan, this weight will be counterpoised by placing the 10

mg. rider at division mark 1 on the beam, since, by the law of the lever,

$$\text{Wt.} \times \text{Wt.-arm} = \text{Load} \times \text{Load-arm}.$$

Substituting in this equation, we have,

$$1 \times 10 = 10 \times 1.$$

(4) Weigh other small objects, such as bits of paper, a match, a dime, etc.

Record all determinations in a neat tabular form.

ERRORS.—Mention at least three sources of error involved in this exercise.

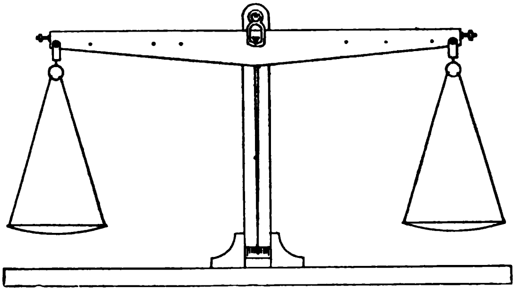

FIG. 13.—Experimental beam balance.

NOTE.—In case one is not so fortunate as to possess a good beam balance, the work in weighing should not be omitted on this account. With the simple and cheap experimental balance which is shown in Fig. 13, all the work laid out in weighing in this manual can be done in a very satisfactory manner. These balances may be home-made at small expense.

EXERCISE 9

Weight of One Cubic Centimeter of Pure Water near the Temperature of Maximum Density

APPARATUS.—A hydrometer jar.* Inside calipers. A meter-stick. A thermometer. A balance and set of weights.

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* A glass tube of about 2 cm. bore, closed at one end with a rubber stopper, will answer as well.

Directions.—(1) See that the hydrometer jar is perfectly dry, inside and outside. Weigh the jar with great care.

(2) Measure the internal diameter of the jar, in the manner shown in Fig. 4. Make several determinations.

(3) Fill the jar nearly full of ice-water. Measure the height of the water column, making an allowance for the curvature of the bottom of the jar. In determining the height of the water column, avoid the error of parallax. Do not read to the edge of the meniscus; always determine the height of a column of liquid in the manner shown in Fig. 14.

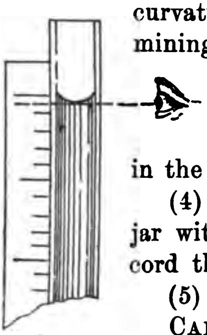

Fig. 14.

(4) Now, find the weight of the hydrometer jar with its contents, in grams. Take and record the temperature of the water.

(5) Repeat (1), (2), (3), and (4).

Calculations.—Compute the volume of the water column, in cubic centimeters. From this, and the weight of the water, compute the weight of 1 cc. of water.

Conclusions.—From tables of water densities get the weight of 1 cc. of pure water at the temperature observed as directed in (4), above. If there is much discrepancy between this weight and that as determined by you, account for it. Mention the chief causes which may have conspired to make the error of your result quite large.

Compute the per cent of error on the basis of the weight of 1 cc. of pure water, at the observed temperature, as given in accepted tables.

Queries.—At what temperature does water attain its maximum density? What is the density of water at this temperature? What is the weight of 1 cc. of water at this temperature? What distinction do you make between the second and third query?

EXERCISE 10

The Beam Balance: Double Weighing and Weighing by Substitution

Apparatus.—Same as that of Exercise 8.

Preliminary Discussion.—The *theoretical* balance has equal arms. *In fact*, however, the arms of no balance are equal. The indications of such a balance must, of course, be incorrect in some degree. Nevertheless, correct weight may be determined with such a balance by the two above-named methods, assuming that the weights are correct. Determined by the method of double weighing, M, the true weight, is given by the equation

$$M = \sqrt{W W_1},$$

where W is the weight required to balance the body when it is in one pan, and W_1 is that required when it is in the other pan.

Directions.

Part A.—**Double weighing.**

(1) Weigh some object, first in one pan, then in the other. Make several determinations, recording the results of each.

Part B.—**Weighing by substitution.**

(1) Place the object used in Part A in the *right-hand* pan. Place enough fine shot in the other pan to counterpoise the object.

(2) Allowing the shot to remain in the pan, remove the object and counterpoise the shot with weights. Barring three small sources of error, you now have the true weight of the object. Make several determinations by this method and record the results of each.

Record all results in a neat tabular form which shall contrast the results obtained by the two methods.

Calculations.—Substitute in the equation and solve for the true weight by the method of double weighing. Compare the results obtained by these methods.

QUERY.—What are the sources of error referred to in (2) of Part B?

EXERCISE 11

The Beam Balance: Weighing by the Method of Oscillations *

APPARATUS.—Same as that of the preceding exercise.

PRELIMINARY DISCUSSION.—The student has already observed that too much time is consumed in waiting for the beam to come to rest in order to determine the point of rest. In addition to overcoming this difficulty, the method of this exercise is valuable in that it is more accurate than any that have preceded it.

Furthermore, the student has observed how difficult it is to secure an exact balance. Fortunately, when this method is used, it is not necessary to produce perfect equilibrium.

DIRECTIONS.—(1) In this method the determination of weight is based upon three positions of rest of the pointer. Each of these points of rest must be determined separately. They are found by observing the turning-points of the pointer on the scale. The *mean* of the turning-points is the point of rest. But, on account of the resistances which oppose the vibration of the beam, the amplitude of these vibrations becomes smaller at each swing. Consequently, the true point of rest is *not half-way* between any two successive turning-points.

For convenience of reference let us call the position of the pointer, when there is no load in the pan, the *first*

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* Thwing's Elements of Physics, p. 229. Stewart and Gee, vol. i, p. 76.

point of rest. To find the first point of rest, proceed as follows:

Take an odd number of turning-points—for example, three to the right and four to the left. Suppose the pointer to move over a scale which is numbered from 0 at the left to 20 at the right—10 being the middle point of the scale. Suppose, further, that the following sets of turning-points are observed:

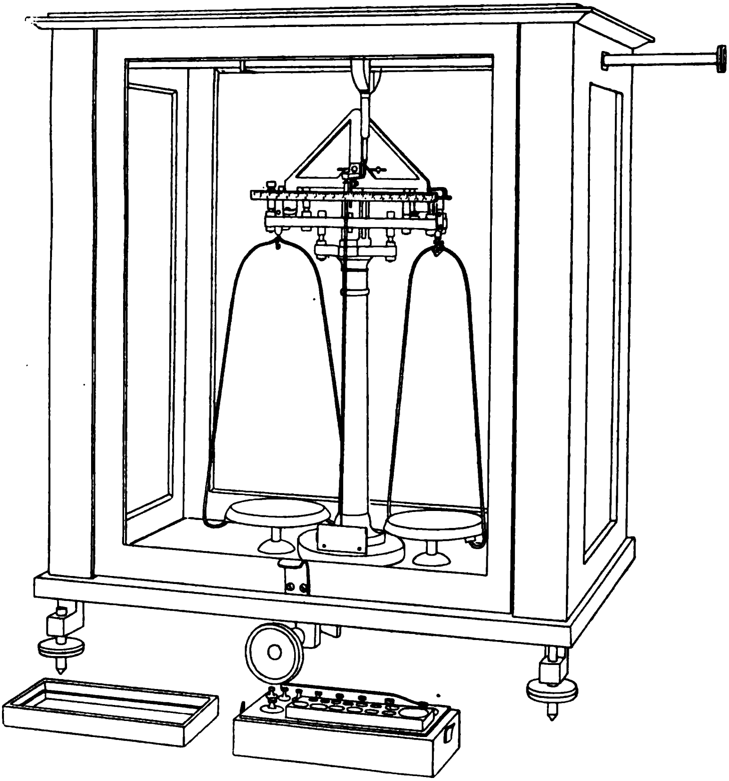

Fig. 15.—Precision balance.

	Left.		Right.
	5.8		13.3
	6.2		13.2
	6.5		12.8
	6.7		. . .
	4)25.2		3)39.3
Mean,	6.3	Mean,	13.1

Whence P_1, the first point of rest, is

$$\frac{6.3 + 13.1}{2} = 9.7.$$

(2) Having found the first point of rest in this manner, place the object in the balance. When approximate equilibrium has been obtained, record a new set of turning-points from which to compute P_2, the second point of rest. (Do not compute the points of rest during the progress of the experiment.)

(3) Now, add 1 cg., by means of the rider, or by placing a weight of this denomination in the pan. Record a third set of turning-points from which to compute P_3, the third point of rest.

Record data in the following form:

Point of Rest.	Turning Points.		Calculated Point of Rest.
	Left.	Right.	
(1)	....	....	P_1 ....
(2)	....	....	P_2 ....
(3)	....	....	P_3 ....

Calculations.—From the sets of turning-points compute the three points of rest. Substitute in the equation,

$$M = W + \frac{P_2 - P_1}{P_2 - P_3} \text{ cg.},$$

in which M is the true weight of the object, W is the total weight in the pan, excluding the weight last added, and P_1, P_2, and P_3 are the first, second, and third points of rest, respectively.*

EXERCISE 12

Graphic Representation of Results: Relation of Volume and Weight of Different Masses of the Same Substance

Apparatus.—A beam balance. A set of weights. Bicycle balls of different sizes. A micrometer caliper.

Preliminary Discussion.—In introducing the student to the subject of plotting, it is desirable that the relation to be represented should be as simple as possible. Let it be required, therefore, to express by means of a curve the relation which exists between *any volume* of a given specimen of bicycle-ball steel and *the weight of an equal volume* of the same specimen.

Directions.—(1) Measure the diameter of each ball in several different diameters.

(2) Weigh each ball several times.

Record each determination.

Calculations.—Compute the mean volume and weight of each ball, and record these mean values in the following form:

	Volume.	Weight.
1st ball	....	....
2d ball	....	....
3d ball	....	....
4th ball	....	....

Etc., etc., etc.

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* In case a milligram weight is added, as directed in (3), above, the equation should read: $M = W + \frac{P_2 - P_1}{P_2 - P_3}$ mg.

Plotting the Results of the Exercise.—Whenever we are dealing with related quantities, both the nature of the relation and the successive values of the quantities themselves may be represented by means of a curve. These curves are very useful in many ways, and particularly so in the respect that a conception of many mathematical relations is materially strengthened by a visualization of such relations through the eye.

Of the many systems of coordination, the system of rectilinear coordinates is perhaps the simplest. Curves plotted according to this system are drawn on paper, which is divided into small squares—preferably into millimeters or tenths of inches—with every fifth and tenth line printed heavier than the intervening lines.

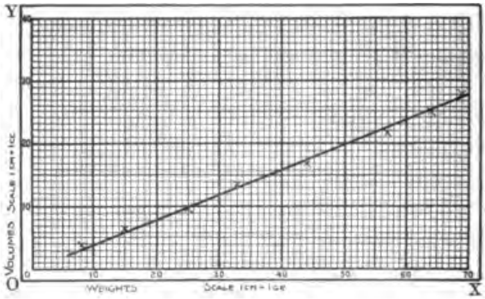

Fig. 16.

To plot a curve based upon your summary of results, proceed as follows:

(1) Select a corner of the sheet of cross-section paper, usually the lower left-hand one, for the origin, calling it O. The sides of the sheet are rectilinear, and are known as the axes: OX, the horizontal line, being known as the axis of abscissas, and OY, perpendicular to OX, as the axis of ordinates.

(2) Using a convenient scale, lay off on OX distances to represent the several weights given in the summary.

Call these points a, b, c, d, etc., respectively. Likewise, on OY, lay off distances to represent the several volumes, calling these points a_1, b_1, c_1, d_1, etc., respectively.

(3) Now, with a as a center, and Oa_1 as a radius, describe a small arc above a; also, with a_1 as a center and Oa as a radius, describe a small arc to the right of a_1. The point of intersection of these arcs is the first point in the required curve.

(4) In like manner, locate points corresponding to b and b_1, c and c_1, d and d_1, etc., respectively.

(5) Finally, connect the points thus determined by a smooth curve—so called, *whether the line is straight or curved.* By a smooth curve is meant a line which passes through, or near, most of the points. Do not draw a broken line from point to point.

RESULTS.—Should a point fall very far out of the curve, such fact discredits the data from which the position of the point was determined. Thus, the curve is a ready means of detecting, or at least of indicating, errors of various sorts. In fact, the curve enables us to *detect* and *interpret* many relations which otherwise would entirely escape our notice.

QUERIES.—The quantity A varies directly as B: What kind of a curve will satisfy this relation? Again, A varies inversely as B: What curve will satisfy this relation? A^2 varies as B^2; A^2 varies inversely as B^2: What curves will satisfy these relations? Could you find the weight of a bicycle ball of known volume simply by inspection of the curve you have drawn, provided the given value lies within the limits of the curve?

EXERCISE 13

Sensibility of a Balance *

APPARATUS.—A beam balance. Accurate weights. An object to be weighed. Cross-section paper.

PRELIMINARY DISCUSSION.—The student will remember that in Exercise 11 a centigram weight was added in order to find a third point of rest from which to compute the entire weight required to bring the pointer to the first point of rest. Now, the labor of making the third determination in each weighing may be avoided by knowing, for every 10 grams of load, within the range of the balance, the number of scale divisions through which the point of rest is moved by the addition of 1 cg. or 1 mg. For example, the sensibility of a certain balance, under a load of 200 grams, is 1.8 divisions. This means that, with 200 grams in each pan, the addition of 1 cg. to one pan will cause the position of rest to be moved 1.8 divisions. If a table of sensibilities is prepared and preserved, much time may be saved in future weighing.

DIRECTIONS.—(1) Determine the sensibility of the balance for every 10 grams of load up to the maximum load of the balance. Take the mean of several determinations as the true value for each load.

Record the data in the following form:

Balance No. ....

LOADS.	Sensibilities.
0	....
10	....
20	....
30	....
40	....
50	....
Etc.	

---

* Stewart and Gee, vol. i, p. 81.

Results and Conclusions.—Compute the sensibility for each load used in the exercise.

Using loads as abscissas and sensibilities as ordinates, plot a sensibility curve for your balance. Paste this curve in your note-book.

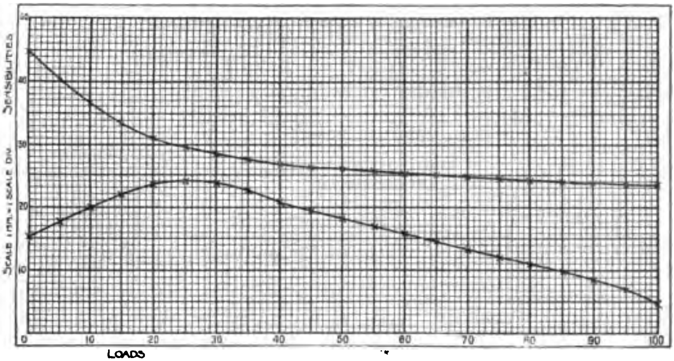

Fig. 17.

In your report give a full and explicit statement of the meaning of a sensibility curve and the use to which it may be put. Before doing so, carefully study the curves given in Fig. 17. These curves represent the varying sensibility of the same balance under different conditions as to adjustment of the instrument. The upper one shows a maximum sensibility with a load of zero. The other one shows a maximum sensibility with a load of 25 grams. These curves seem to indicate that the point of maximum sensibility may be changed to some extent by adjusting the instrument. What are the adjustments referred to? Can any practical use be made of our ability to change the point of maximum sensibility?

CHAPTER II

ELASTICITY OF SOLIDS

EXERCISE 14

Tenacity of Wires

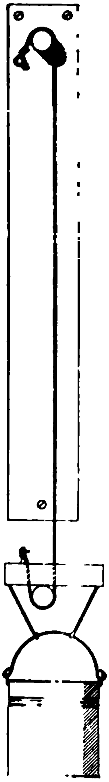

FIG. 18.

APPARATUS.—Samples of steel, iron, and copper wires. A micrometer caliper. A pail of suitable size to attach to the lower end of the wires. The wires should be attached to the support and to the pail in the manner suggested by Fig. 18.

PRELIMINARY DISCUSSION.—As used in physics, the word tenacity has a technical, as well as a practical, significance. Technically considered, the tenacity of a body of uniform cross-section is its breaking strength in dynes per square centimeter of cross-section. In a practical way, the tenacity of a body of uniform cross-section is its breaking strength in pounds or kilograms per unit of cross-section. (Why specify uniform cross-section in both definitions?)

DIRECTIONS.—(1) Suspend the pail by the steel wire from a firm support. Underneath the pail place a box containing sawdust to deaden the fall of the pail when the wire breaks.

(2) Place just enough weight in the pail to keep the wire taut. Measure the diameter of the wire in several places with the micrometer caliper.

(3) Now add a weight which will almost break the wire. (The amount of this weight should be given by the instructor.) Add shot, little by little, until the wire breaks.

(4) Weigh the pail with its contents.

(5) Repeat (1), (2), (3), and (4), using iron and copper wires.

Record data in a neat form.

CALCULATIONS AND RESULTS.—Compute the mean diameter of each wire. Then compute the breaking strength of each wire, in kilograms per square centimeter, and in pounds per square inch, of cross-section.

Refer to the technical definition of tenacity given above and frame an equation which shall express its meaning. (Look up the definition of dyne in this connection, if you do not know it.)

Substitute values obtained in the experiment in this equation and solve for the tenacity of each kind of wire. Compare these results with the corresponding breaking strengths in kilograms and pounds. Note the nature of the difference between these results.

EXERCISE 15

Elasticity of Stretching

APPARATUS.—A fine steel or brass wire suspended from a firm support close to a millimeter scale. At a distance of about a meter apart are placed two reading microscopes (ordinary magnifying-glasses will do very well) which are to be used to determine the position of the markers with respect to finely divided scales mounted near the extremities of the meter scale. The lower microscope only is to be used in Part A.

PART A.

PROPOSITION.—**To prove that, within the limits of elasticity, the elongation of a wire is proportional to the stretching force.**

Directions.—(1) Place a kilogram weight in the scale-pan. This weight is to keep the wire taut. Do not remove it throughout the exercise.

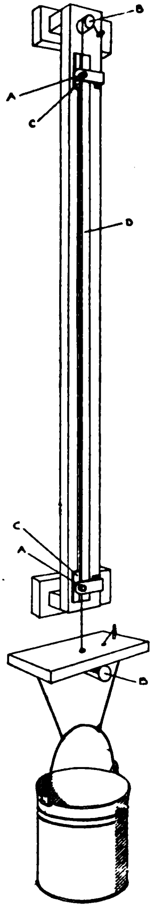

Fig. 19.
A, reading microscopes; *B*, pins; *C*, fine scales; *D*, meter scale.

(2) Tie a hair or a fine silk fiber—to be used as a marker or index—tightly around the wire in the field of view of the lower microscope. Now read the zero position of the index on the scale of the eye-piece, if it is provided with one, or on the fine scale of the apparatus, estimating the fractional division in tenths.

(3) Place 2 kg., or thereabouts, in the scale-pan. Call this weight f, and record its exact value. Take the reading of the index.

(4) Again add 2 kg., and record the total weight, f_1, which, be careful to remember, is the total weight in the scale-pan, excluding the weight mentioned in (1). Take the index reading.

(5) Finally, add 2 kg., if the wire will carry this weight safely. Record f_2 and take the index reading.

(6) Remove all weights with the exception of that mentioned in (1). Observe whether the index returns to the zero point. If it lacks much of doing so, either the wire has slipped in the clamp which holds it, or an error was made in the first index reading.

Record the data in the following form:

Stretching Force.	Index Reading.	Elongation.
0. (No load). f = f_1 = f_2 =		

Calculations and Results.—State, in the form of a proportion, the proposition which is to be verified in this exercise.

In your report give the meaning of this proposition in words which are not a repetition of the proposition itself.

Substitute numerical values in the proportion you have framed, and verify.

Using total elongations as abscissas and stretching forces as ordinates, plot a curve based on the values given in your table of results. (What kind of a curve should it be?)

Part B.

Proposition.—**To determine the stretch modulus of brass or steel wire.**

Preliminary Discussion.*—The stretch modulus is a constant which is used to calculate the ability of a body to resist elongation. Simply stated, the stretch modulus is that force which will stretch a body to double its initial length, provided that, in doing so, the limit of elasticity is not exceeded.

Now, if this modulus be represented by *M*, it can be shown that

$$\mathrm{M} = \frac{\text{stress}}{\text{strain}}. \tag{1}$$

$$\text{But, stress} = \frac{\text{Force applied}}{\text{Area of cross-section}}, \tag{2}$$

$$\text{and strain} = \frac{\text{Increase of length}}{\text{Initial length}}. \tag{3}$$

$$\text{Whence, } \mathrm{M} = \frac{FL}{al}, \tag{4}$$

where F is the stretching force, L is the initial length, l is the increase in length, and a is the area of cross-section.

The stretch modulus, which is sometimes known as

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* Stewart and Gee, vol. i, p. 176.

Young's Modulus, is usually expressed according to the C. G. S. system. In substituting in equation (4), therefore, F should be expressed in dynes, and a, L, and l in centimeters.

Directions.—(1) Measure the diameter of the wire in several places with a micrometer caliper. Record these measurements in a table supplemental to the one given below.

(2) Proceed as directed in (1) and (2) of Part A, using both microscopes. Determine in centimeters the distance, L, between the indices.

(3) Place 6 kg. in the scale-pan. Take the reading of both indices with great care.

(4) Remove the load, with the exception of that noted in (1) of Part A, and observe whether the indices return to the starting-point. (See (6) of Part A.)

(5) Repeat (1), (2), (3), and (4) at least twice. Record data in the form which follows:

Trial.	UPPER INDEX.		
	(1) First position (without load).	(2) Second position (with load).	(3) Amount of lowering in centimeters.
(1)			
(2)			
(3)			

Trial.	LOWER INDEX.		
	(4) First position (without load).	(5) Second position (with load).	(6) Amount of lowering in centimeters.
(1)			
(2)			
(3)			

Calculations.—Compute the mean diameter of the wire used, and therefrom the area of cross-section. The increase of length, l, may be obtained by subtracting the amount of lowering of the lower index from that of the upper index. Substitute each set of values in equation (4) and solve for Young's Modulus.*

EXERCISE 16

The Jolly Balance

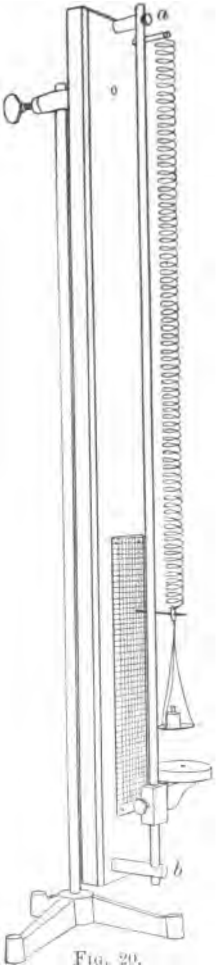

Fig. 20.

Apparatus.—A Jolly balance. A set of metric weights.

Preliminary Discussion. — The Jolly balance is simply a very sensitive form of the ordinary spring balance. The principles of elasticity already developed apply to spring balances. The distortion which the spring of the balance undergoes when it is extended calls into action an elasticity of a mixed nature—being, in fact, an elasticity of bending combined with an elasticity of torsion. The element of torsion is a minor factor in the case, however. The fact that the elongation of the spring is proportional to the stretching force makes it seem advisable to introduce the Jolly balance at this point.

Part A.

Directions.—(1) Read the position of the index, with no load in the pan, avoiding parallax.

(2) Place 5 grams in the pan. Read the position of the index.

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* Stewart and Gee, vol. i, p. 175.

(3) Repeat (1) and (2), with loads of 10, 15, 20, and 25 grams (if the spring will carry this much), reading the position of the index both before and after using each weight.

Record data in the following form:

LOAD.	Reading without load.	Reading with load.	Elongation.	Elongation per gram.
0 grams.				
5 "				
10 "				
15 "				
20 "				
25 "				

PART B.

PRELIMINARY STATEMENT.—It is required to weigh some object with a Jolly balance *without a set of weights.*

DIRECTIONS.—(1) Take the reading of the index with no load in the pan. Place a small object—for example, a bit of paper or a piece of a match—in the scale-pan and take the reading of the index. On removing the object from the scale-pan, observe whether the index returns to the zero position. Make several determinations of the elongation produced by each object.

(2) Place a centigram scale-weight in the pan, and proceed as before.*

(3) Weigh each object with a sensitive beam-balance.

Record data in a neat tabular form.

CALCULATIONS.—Compute the mean elongation of the spring per gram. (Use data obtained as directed in Part A.) Compute the mean elongation produced by each object. Now calculate the weight of each object.

Assuming the weights to be correct, as determined with the beam-balance, compute the per cent of error for each determination.

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* Or a weight suited to the capacity of the spring.

QUERIES.

(On Part A.) Should the results in the last column agree? Did the index return each time to the starting-point? Would it have continued to do so if you had increased the load indefinitely? How would you explain a failure to do so? Are the scale divisions of a common spring balance of equal size? Why?

(On Part B.) How do the makers of spring balances graduate these instruments?

EXERCISE 17

Elasticity of Bending

APPARATUS.—A well-seasoned wooden bar supported by three-cornered blocks, as shown in Fig. 21. A scale-pan of some sort is suspended from the middle of the bar. A pointer, moving through a wide range over a scale placed about a foot from the bar, is to be used to indicate the deflections of the bar.

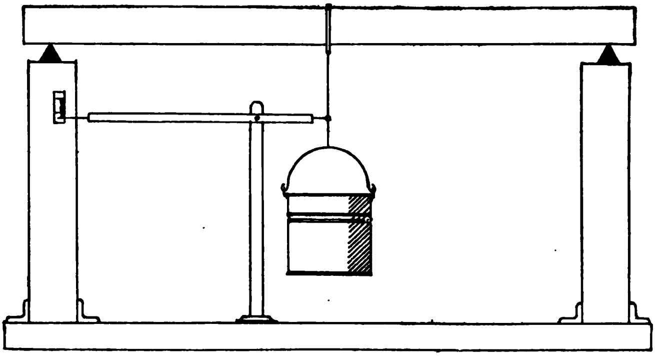

FIG. 21.

PROPOSITIONS. PART A.—(1) **The deflection of a rod or beam is proportional to the deflecting force when the length, breadth, and thickness of the rod or beam are constant.**

(2) The deflection of a rod or beam is proportional to the cube of the length of the rod or beam when the deflecting force and the breadth and thickness of the rod or beam are constant.

DIRECTIONS.—(1) Place the three-cornered supports about 80 cm. apart. Call this length l. See that the supports are as nearly parallel as possible. Place the bar squarely on the supports. With no load in the scale-pan, read the position of the pointer on the scale.

(2) Place a load of 200 grams in the scale-pan. Call this weight F. Take the reading of the pointer on the scale.

(3) Remove the weight from the scale-pan and observe whether the pointer returns to the position it occupied when no load was in the pan. If it does not, repeat (1) and (2).

(4) Place 500 grams in the pan. Call this weight F_1. Take the reading of the pointer.

(5) Place the supports 60 cm. apart. Call this distance l_2. Place 500 grams in the pan. Call this weight F_2.

Record data in the following form:

Readings of pointer.
First, or zero............
Second.................
Third..................
Fourth.................

DETERMINATION.	Length.	Deflection.	Deflecting force.
(1)	$l =$	$d =$	$F =$
(2)	$l_1 =$	$d_1 =$	$F_1 =$
(3)	$l_2 =$	$d_2 =$	$F_2 =$

CALCULATIONS AND RESULTS.—From your table of pointer readings determine the number of scale divisions through which the outer end of the pointer was moved by each load used. These distances, or scale divisions, may represent the respective deflections, d, d_1, and d_2. Why?

Frame a proportion which shall express proposition (1).

Frame a proportion which shall express proposition (2).

Substitute in these proportions, and verify.

Include in your report, in the simplest and most direct language you can use, a concise statement of the meaning of the above propositions.

PART B.—**Alternative method.**

APPARATUS.—Two well-seasoned wooden bars, supported on knife edges. A metal pin projects above the surface of each bar, about the middle. A spherometer is mounted on a firm support and is so placed that the screw of the spherometer is directly over the pin just referred to.

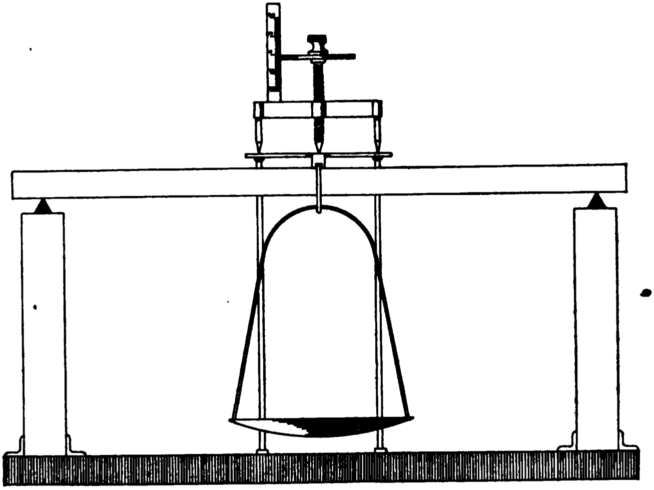

FIG. 22.

PRELIMINARY DISCUSSION.—In Part A the breadth and thickness of the bar were not considered. It is the purpose of Part B to take account of these factors.

PROPOSITION.—**When a bar is bent within the limits of elasticity the deflection is directly proportional to the de-**

flecting force and to the cube of the length, and inversely proportional to the breadth and to the cube of the thickness.

DIRECTIONS.—(1) Using the narrow bar, proceed as directed in (1) and (2) of Part A, with the exception that readings of the spherometer are to be taken in place of pointer readings.* Measure the length of the bar with great care. Measure the breadth and thickness of the bar at several places. Record all measurements and take their mean at the proper time.

(2) Adjust the apparatus to use the broader bar. Use a load of 500 grams. Take spherometer readings as directed in (1), above. Measure the length, breadth, and thickness of the bar.

Record data in the following form:

SPHEROMETER READINGS.	Linear scale.	Circular scale.
First, or zero. Second. Third.		

BAR.	Load, F.	Length, l.	Breadth, b.	Thickness, t.	Deflection, d.
(1)	....	....	.... *Mean*, ..	.... *Mean*, ..	....
(2)	....	....	.... *Mean*, ..	.... *Mean*, ..	....

CALCULATIONS AND RESULTS.—From the readings of the spherometer calculate the deflection for each load.

---

* A reading of the spherometer should be taken before a load is placed in the pan.

Record these deflections in your table of results. Compute the mean breadth and thickness of both bars.

Frame a proportion which shall express the law of bending given under Preliminary Discussion of Part B. Substitute numerical values in this proportion, and verify.

PROBLEM.—Suppose a pine timber, 2 inches thick and 4 inches wide, carries with safety a load of 100 pounds at its middle point when the distance between the supports is 8 feet. How great a load can a similar pine timber carry if its thickness is 12 inches, its width is 8 inches, and the distance between the supports is 12 feet, assuming both timbers to be of similar shape and material, and a deflection of 0.25 inch to be permissible in both cases?

EXERCISE 18

Elasticity of Torsion

APPARATUS.—Two brass rods, approximately of the dimensions indicated in the problem given below. These rods are to be used, one at a time, in the apparatus shown in Fig. 23. If the bearing in the right-hand upright is properly made and lubricated, the apparatus may be made to yield excellent results. A simpler though less satisfactory form of the apparatus is shown in Fig. 24.

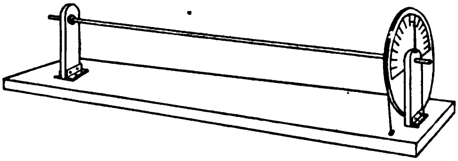

FIG. 23.

PROPOSITION.—Suppose a rod to be fixed at one end and a twisting force to be applied at the free end. Let it be required to prove:

The angle through which the free end of the rod may be twisted is *directly* proportional to the twisting force and to the length of the rod, and *inversely* proportional to the fourth power of the diameter of the rod.

DIRECTIONS.—(1) Use a weight which will twist the smaller rod through a moderately large angle. (The instructor should suggest the magnitude of this angle.) Call the twisting force f, and the angle of torsion T. Measure the length of the rod.* Call it l. Measure the diameter, d, of the rod in several places with a micrometer caliper. Repeat all these operations at least twice.

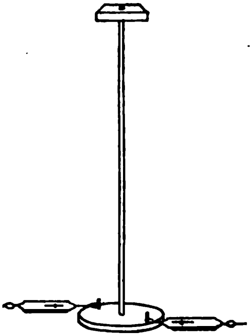

FIG. 24.

(2) Remove this rod from the apparatus and substitute the larger one. Place the pulley so that the length of the rod, as already defined, shall be different from its former value. Proceed as directed in (1), above, calling the angle of torsion T_1, the twisting force f_1, the length l_1, and the diameter d_1. Repeat as before.

Record linear dimensions in centimeters. Use the following form:

ROD.	Trial.	Twisting force.	Length.	Diameter.	Angle of torsion.
Larger	1 2 3 4 Etc.	*Mean,*	*Mean,*	*Mean,*	*Mean,*
Smaller	1 2 3 4 Etc.	*Mean,*	*Mean,*	*Mean,*	*Mean,*

---

* This length is the distance from the fixed end of the rod to the point at which the angle of torsion is determined.

Calculations and Results.—Compute the mean values called for in the tabular form.

Frame a proportion which shall express the proposition to be proved in this exercise.

Substitute mean numerical values in this proportion, and verify.

Problem.—A brass rod having a diameter of 0.6 cm. is twisted through an angle of 16° by a force of 6 kg. applied 46 cm. from the fixed end of the rod. What force will be necessary to twist a brass rod of similar shape and material through an angle of 8°, if the diameter of the rod is 1.2 cm. and its length is 90 cm.?

CHAPTER III

MECHANICS OF SOLIDS

EXERCISE 19

Composition of Parallel Forces

PART A.

APPARATUS.—A meter-stick suspended at its extremities by two spring balances. A weight suited to the capacity of the spring balances.

PRELIMINARY DISCUSSION.—By composition of parallel forces is meant the *process* of finding a single force which is equivalent to the combined effect of the parallel forces. Each of the given parallel forces is called a component force. The single force which is equivalent to these forces is called the resultant.

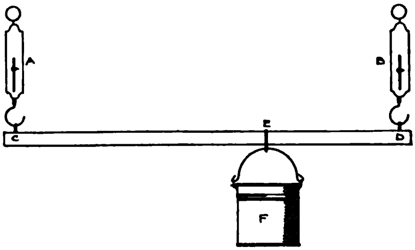

FIG. 25.

DIRECTIONS.—(1) Suspend the balances at the same height and see that the meter-stick is practically horizontal.

(2) Call the left-hand balance A and the right-hand one B. Call the points on the meter-stick at which A, B, and the weight, F, are attached, C, D and E, respectively.

(3) Take the index readings of A and B before F is suspended from the meter-stick. These will be the zero readings. (Why is it necessary to take zero readings?)

(4) Locate E at unequal distances from C and D. Record the amount of weight, F, and the index readings of A and B. Measure CE and DE.

(5) Repeat (4) four times, using different distances each time.

Record the data in the following form:

Trials.	Reading of A.	Reading of B.	CE.	DE.	A × CE.	B × DE.
0 (zero)						
(1)						
(2)						
(3)						
(4)						
(5)						

Results.—Show, by the principle of moments, that the following equations are true:

(1) $A \times CE = B \times DE.$

(2) $F \times CE = B \times DC.$

(3) $F \times DE = A \times DC.$

(4) $F \times DC = (A \times DC) + (B \times DC).$

Part B.—**Alternative method.**

Apparatus.—A home-made apparatus of the form suggested by Fig. 26 may be constructed which will give results much more accurate than those obtained by the use of spring balances. By careful counterpoising before be-

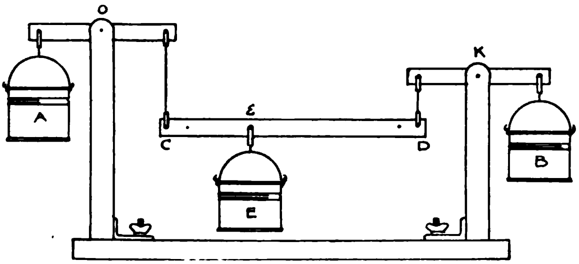

Fig. 26.

ginning the exercise proper, the weight of the levers may be almost wholly eliminated.

PROPOSITION.—Let it be required to prove the truth of the following equations:

$$A \times Ce = B \times De.$$
$$E \times Ce = B \times DC.$$
$$E \times De = A \times DC.$$
$$E \times DC = (A \times DC) + (B \times DC).$$

PRELIMINARY DISCUSSION.—The effect of the weights placed in A and B is to produce forces which act in an upward direction at C and D, respectively. When a weight is hung from the bar at e, that point becomes the point of application of the resultant of the three forces due to the weights in A, B, and E, respectively. The total weight at e is also the equilibrant of A and B, and is, of course, equal to their resultant.

DIRECTIONS.—(1) Before suspending E from the bar, counterpoise the bar by placing sand or fine shot in A and B. When this has been done as perfectly as possible, all the levers should be sensibly horizontal. The levers should not bind at O and K. The success of this method depends in large measure upon this particular point.

(2) Suspend E from the bar at unequal distances from C and D. Place a small light vessel in both A and B—e. g., a small tin cup. Place a kilogram weight in E. Now, pour fine shot or sand into the tin cups in A and B until the system of levers is in perfect equilibrium.

(3) Remove E and the tin cups from the apparatus. Determine with great care the weight of each. Let A represent the weight of the cup and shot removed from pail A, and B that of the cup and shot removed from pail B.

(4) Repeat (1), (2), and (3).

Record data in a form similar to the one suggested in Part A.

RESULTS.—Substitute numerical values in each of the equations given above, and verify.

EXERCISE 20

Resultant of Three Forces Concurrent in One Plane

Apparatus.—A resultant of forces apparatus of the Nichols form, or, in the absence of one, the apparatus shown in Fig. 28 will answer fairly well.*

Directions.—(1) Place weights of such magnitude in the scale-pans as to cause the cords *AO* and *BO* to form at *O* the angle which is shown in the figure. (An approximation to this angle is all that is meant.)

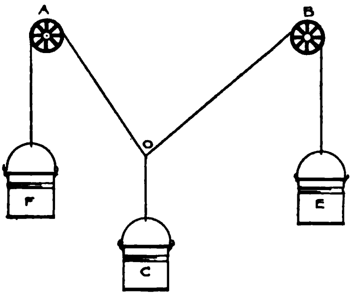

Fig. 27.

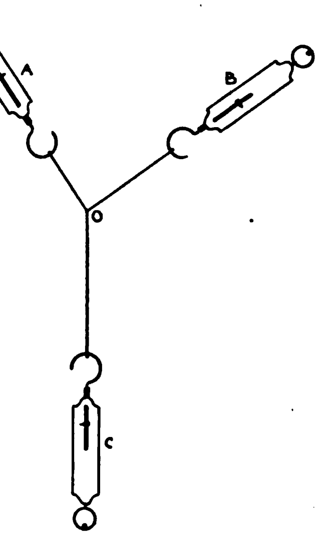

Fig. 28.

(2) With a sharp pencil, held perpendicular to the paper, trace the angles formed by the cords at *O*. (Each student must make a tracing.)

(3) Remove the scale-pans. Weigh each with its contents. These weights are the forces which act in the direction of *OC*, *OB*, and *OA*, respectively.

(4) Repeat (1), (2), and (3), using weights of such mag-

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* See Nichols's Laboratory Manual, vol. i, p. 40.

nitude as to form at *O* angles different from those produced as directed in (1), above.

(5) In case apparatus of the Nichols form is available, arrange the apparatus to use four and five concurrent forces, acting as shown in Fig. 27. Proceed to make tracings in each case, as directed in (2), above.

Record data in a neat form.

GRAPHICAL DETERMINATION OF THE RESULTANT

The weights at *C*, *E*, and *F* act at the point *O*, in the direction of *OC*, *OB*, and *OA*, respectively. Now, if the magnitude of these forces be represented on the corresponding lines, their resultant may be determined graphically.

Using any convenient scale, from *O* on *OC*, on *OB*, and on *OA*, lay off distances to represent these forces. Draw *OH*, the resultant of *OB* and *OA*. Determine the numerical magnitude of *OH* by scale.

Now, the force *C* balances that of *E* and *F*. That is, *OC* must be equal to the resultant of *OB* and *OA*, and opposite to it. Since *OC* is equal and opposite to *OH*, the resultant of the three forces must be zero. This reasoning may be extended to a similar case wherein any number of forces is involved.

Observe, finally, that the critical test to be applied in proving that the resultant of several forces is equal to zero, consists in proving that *any one* of them is equal to the resultant of all the rest, and opposite thereto.

EXERCISE 21

On Machines: Law of the Lever

APPARATUS.—A lever provided with scale-pans whose points of suspension may be located at pleasure along the lever. A balance and a set of weights.

PRELIMINARY DISCUSSION.—In this exercise the weight of the lever need not be considered, since it is balanced before the scale-pans are attached.* In the next exercise, however, the lever will be treated as an actual, not an ideal, one.

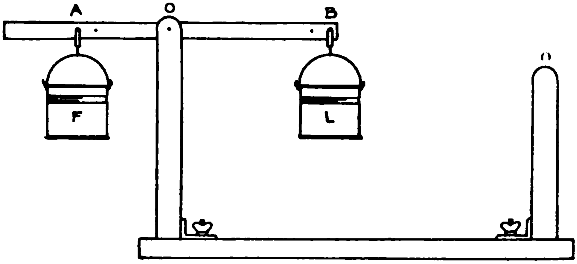

FIG. 29.—Lever of first class.

A lever may be used in three different ways, as shown in Figs. 29, 30, and 31.

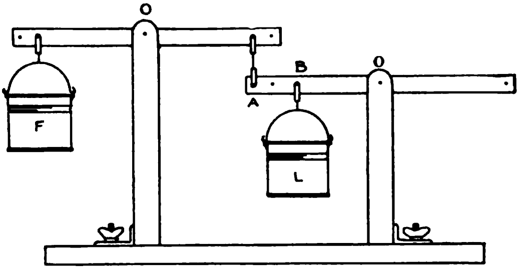

FIG. 30.—Lever of second class.

In each figure, let A and B be the point of application of the force and the load, respectively, and O the fulcrum. Then AO is known as the *arm of the force*, and BO as the *arm of the load.*

---

* Before performing the operations indicated in (1), (3), and (4), the lever or combination of levers *must be balanced.* This may be done conveniently by means of riders of wire which may be shifted along the levers.

We may now state the fundamental law of all machines, in two ways, as follows:

(1) Force × Its Arm = Load × Its Arm.

(2) Force × Distance Force Moves = Load × Distance Load Moves.

Directions.—(1) Arrange the lever so that *O* shall be between *A* and *B*, but nearer to one than to the other. Place a small weight in one scale-pan. Counterpoise with shot and bits of paper. Carefully weigh each scale-pan with its contents.

(2) Measure *AO* and *BO* in centimeters.

(3) Arrange the apparatus as shown in Fig. 30. Proceed as directed in (1) and (2).

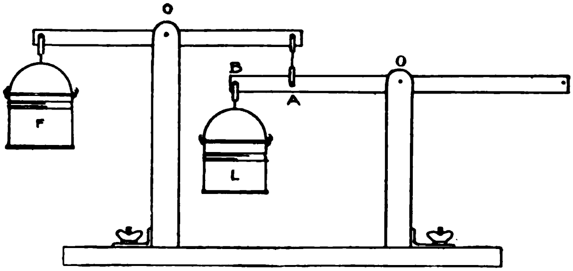

Fig. 31.—Lever of third class.

(4) Arrange the apparatus as shown in Fig. 31. Proceed as directed in the latter part of (1) and (2).

Tabulate results in the form which follows:

Relation.	Force.	Force-arm.	Load.	Load-arm.
(1)				
(2)				
(3)				

Calculations.—Substitute each set of values in equations (1) and (2), and verify.

EXERCISE 22

On Machines: The Practical Lever

APPARATUS.—A rectangular wooden or iron bar, 4 feet long. A triangular block to be used as a fulcrum. Two improvised scale-pans. Weights of such magnitude as will not sensibly bend the bar.

Let O be the fulcrum and A and B the points of suspension of the force and load, respectively.

DIRECTIONS.—(1) Make several measurements of the breadth and thickness of the bar, to make sure that it is approximately rectangular.

(2) Weigh the bar. Measure its length with great care.

(3) Arrange the apparatus as shown in Fig. 29. Place a weight in scale-pan L. Counterpoise the weight which is suspended from A with weights and shot suspended from B.

(4) Determine the total force applied at A and the total load applied at B. Measure the arm of the force and that of the load.

(5) With O at other positions, repeat (3) and (4).

Record data in a neat tabular form.

RESULTS.—It is evident that the total *clockwise* moment involved in this exercise must consist of two parts: (1) The moment of the load. (2) The moment due to the weight of the part of the bar $B\,O$. In like manner, the *counter-clockwise* moment about O must consist of the moment of the force and that due to the weight of the part of the bar $A\,O$. But the algebraic sum of these moments is zero—that is, calling clockwise moments positive, and counter-clockwise moments negative, we have,

$$\textit{Mom. of Load} + \textit{Mom. of } BO - \textit{Mom. of Force} - \textit{Mom. of } AQ = 0.$$

Calculate the numerical value of the moments designated in the equation given.

Substitute these values in the equation, and verify.

Problem.—A lever has a uniform cross-section and is used as a lever of the first-class. The arm of the force is 4 feet. The load to be lifted is 200 pounds, and the load-arm is 8 inches. Taking the weight of the lever, 8 pounds to the running foot, into account, what additional force, if any, must be employed to counterpoise the load?

EXERCISE 23

On Machines : The Pulley

Apparatus.—Sets of single and double pulleys. The cords which pass over the pulleys are provided with scale-pans. The one in which a lesser weight is to be placed may be known as F and the other as L.

Part A.—**Combination of a double fixed pulley and a single movable pulley.**

Directions.—(1) Place a kilogram weight in L. Put just enough fine shot or sand in F to cause it to descend very slowly. Weigh the scale-pan with its contents. Call this weight F_1. Replace the scale-pan. Remove just enough shot to cause the pan to rise very slowly. Weigh the pan with its contents. Call this weight F_2. Then the true force, F, is given by the simple equation,

$$F = \frac{F_1 + F_2}{2}. \quad \text{(Why?)}$$

(2) Weigh the total load, L, including movable pulley.

(3) Next determine how far, in centimeters, F moves when L moves 15 cm. Let D be the distance F moves, and D_1 the distance L moves.

(4) Repeat (1), (2), and (3), using different loads.

Part B.—**Combination of a double fixed pulley and a double movable pulley.**

Directions.—(1) Arrange the apparatus as shown in Fig. 33.

(2) Proceed as directed in (1), (2), and (3) of Part A.

Record data in the following form:

TRIAL.	F_1.	F_2	F.	L.	D.	D_1.
(1)						
(2)						
(3)						
Etc.						

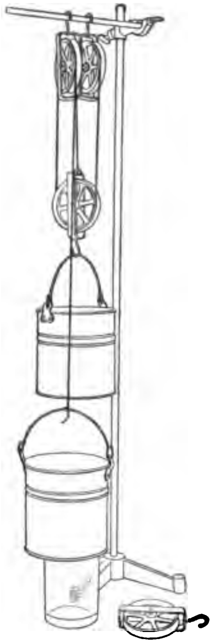

FIG. 32.

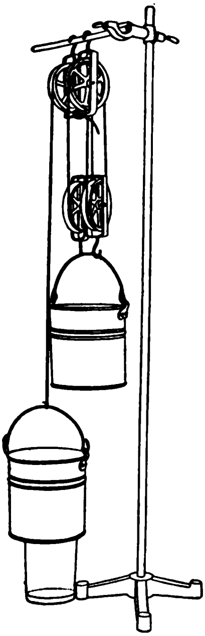

FIG. 33.

CALCULATIONS AND CONCLUSIONS.—Make a simple rule for determining the advantage to be derived from any combination of pulleys, neglecting friction.

Substitute each set of values in the general law of machines, and verify.

EXERCISE 24

On Machines: The Inclined Plane

Apparatus.—An inclined plane, adjustable to different angles. A car with good bearings. A balance. A set of weights.

Directions.—(1) Adjust the inclined plane to an angle of 20°.

(2) Place about 100 grams in the car. Weigh the car with its contents, and call this weight the load, *L*. Now put just enough sand into the scale-pan to cause the car to move slowly up the inclined plane. Weigh the scale-pan with its contents, calling this weight F_1. Replace the pan. Remove just enough sand to cause the car to move slowly down the incline. Weigh as before, calling this weight F_2. Then the true force, *F*, is given by the equation,

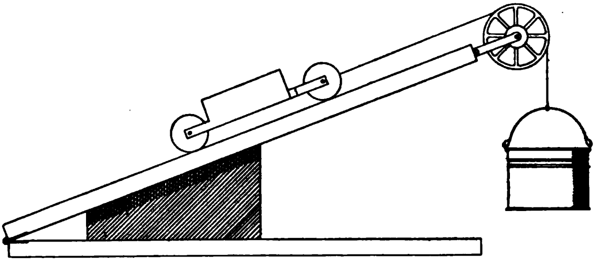

Fig. 34.

$$F = \frac{F_1 + F_2}{2}.$$

(3) Measure the length, *D*, of the inclined plane. This is the distance through which the force must move in order to lift the load to the top of the incline. Measure the vertical height of the inclined plane, D_1. (Disregarding friction, it requires *no work* to move the car in a horizontal

direction, the work being entirely expended in lifting the car through a distance equal to the height of the inclined plane. Therefore, D_1, the distance the load moves, is the vertical height of the inclined plane). Repeat (1), (2), and (3) several times.

(4) Repeat (2) and (3), using an angle of 45°. Record data in the following form:

ANGLE.	Trial.	F_1.	F_2.	F.	D.	L.	D_1.
30°	(1) (2) (3) Etc.						
45°	(1) (2) (3) Etc.						

CALCULATIONS.—Compute mean values of F, L, D, and D_1. Substitute these values in the general law of machines, and verify.

QUERIES.—Why can a person move a very heavy weight to a considerable height by means of an inclined plane? How does he obtain the advantage which enables him to move the weight?

EXERCISE 25

On Machines: The Wheel and Axle

APPARATUS.—A wheel-and-axle apparatus. A meter-stick. Outside calipers.

DIRECTIONS.—(1) Place several kilograms in the scale-pan attached to the cord which passes over the smaller wheel, or axle. Call this weight L. Proceed, as directed in Exercise 23, to find the true force, F, which will balance this load. Measure the distance through which F moves while

L moves a given distance, say 4 dm. Call the former D, and the latter D_1. Measure with the outside calipers the diameter of the wheel and that which serves as the axle. Call the radius of the wheel R, and that of the axle R_1.

(2) Repeat (1) several times, using different weights.

Record data in the following form:

TRIAL.	F_1.	F_2	F.	L.	D.	D_1.	R.	R_1.

RESULTS.—Compute mean value of F, L, D, D_1, R, and R_1. Substitute mean values in the general law of machines, and verify. Substitute in the equation,

$$FR = LR_1,$$

and verify.*

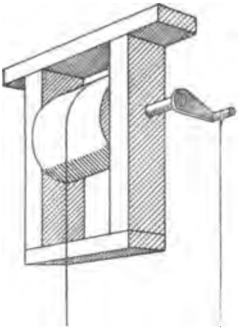

FIG. 35.

QUERIES.—Why is the crank of a derrick-windlass made long? Why did you measure R and R_1? Why can an enormous weight be lifted by a single man by means of a wheel and axle (or windlass) properly constructed? (Be clear and definite in your answers.)

---

* Show that this equation is equivalent to the general law of machines.

EXERCISE 26

On Machines: Starting and Sliding Friction

APPARATUS.—The inclined-plane apparatus used in Exercise 24. A meter-stick. A balance and set of weights.

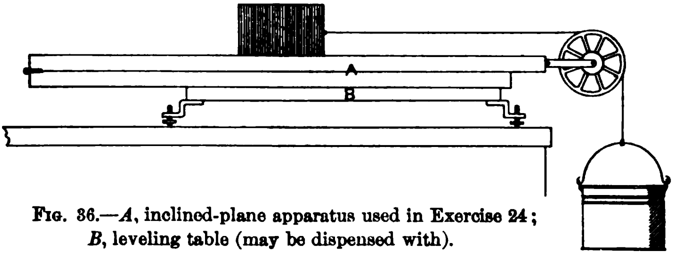

FIG. 36.—*A*, inclined-plane apparatus used in Exercise 24; *B*, leveling table (may be dispensed with).

PRELIMINARY DISCUSSION.—When one body is placed upon another, the adhesion between the faces in contact tends to prevent any relative motion of either. A certain and considerable force must be employed to cause one to start to slide over the other. The amount of force necessary to start this motion is greater than that necessary to maintain it.

There is always a definite relation existing between this force and the effective pressure of the upper body against the lower. This relation depends upon the material and condition of the bodies under consideration, and is known as the coefficient of friction.

When F is the force necessary to start, or to maintain, motion of one body upon the other, and P is the effective pressure exerted by the body to be moved, the coefficient of starting friction, as well as that of sliding friction, is expressed by the ratio, $\frac{F}{P}$. When these coefficients are to be determined by the method of the inclined plane, either coefficient, $a = \frac{BC}{AC}$. The truth of this statement will be apparent from a consideration of the following:

Let W be a block and weight which will just start to slide down the inclined plane, AB, when the latter makes a certain angle with the base, AC. (The amount of this angle depends upon the character of the two surfaces in contact, as already stated.) This angle is called the *angle of repose.*

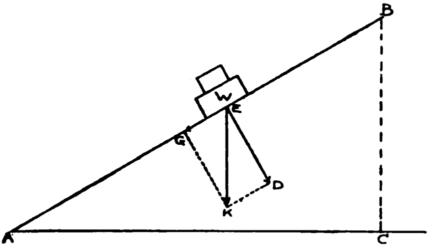

FIG. 37.

The force of gravity acting between the earth and the block in the direction of EK may be resolved into two rectangular components, one of which, ED, perpendicular to AB, produces the pressure against AB, while the other, EG, produces motion in the direction of BA. Now, EG is equivalent to F and ED to P.

But by geometry $\frac{EG}{ED} = \frac{BC}{AC}$.

Whence, a $= \frac{BC}{AC}$.

PART A.—**Coefficient of starting friction.**

(I) *Motion in the horizontal.*

DIRECTIONS.—(1) Place a pine block, of dimensions 2 × 7 × 10 cm. on the board. Both block and board should be freshly sand-papered. Attach a cord to one end of the block and pass it over a pulley which is fastened to one end of the board. Attach a pail to the cord to serve as a scale-pan. Place a 500-gram weight on the block.

(2) Now, little by little, place weights in the scale-pan until the block starts to move. Weigh the scale-pan, with its contents. This weight is the starting force, F. (A further small amount might be added or subtracted — which? — to allow for the friction-loss in the pulley. Determine this loss if you can and allow for it.)

(3) Weigh the block. This weight, plus that of the 500-gram piece, is the pressure, P.

(4) Place the block and weight at other positions on the board, and proceed as directed in (1), (2), and (3).

(II) *Motion on an inclined plane.*

DIRECTIONS.—(1) Place the weight on the inclined plane while it makes but a small angle with its base. Now, slowly and carefully elevate the inclined plane until the weight will just start to slide. Measure BC and AC, as shown in Fig. 37. Repeat as directed in (4), above.

PART B.—**Coefficient of sliding friction.**

(I) *Motion in the horizontal.*

DIRECTIONS.—(1) Proceed as directed in (1), (2), (3), and (4) of (I) of Part A, except that just enough weight is to be placed in the scale-pan to cause the block and weight to move slowly and with uniform motion along the board. While this is being done, the board should be tapped continuously to overcome the starting friction, which, as already noted, is greater than the sliding friction. Repeat.

(II) *Motion on an inclined plane.*

DIRECTIONS.—(1) Proceed as directed in (II) of Part A. The block should move as described in (1) of Part B. The board should be tapped while the weights are being placed in the scale-pan. Repeat.

Record the data in the following form:

PART.	MOTION IN THE HORIZONTAL.			MOTION ON AN INCLINED PLANE.		
	Trial.	F.	P.	Trial.	BC.	AC.
A	(1) (2) (3) (4) Etc.			(1) (2) (3) (4) Etc.		
B	(1) (2) (3) (4) Etc.			(1) (2) (3) (4) Etc.		

CALCULATIONS.—Compute mean values of *F*, *P*, *BC*, and *AC*. Compute the cöefficient of starting and sliding friction by both methods and compare results.

QUERIES.—Which coefficient should be greater? Why? Why is it so difficult to start a heavily loaded train or truck?

EXERCISE 27

The Gravity Pendulum

APPARATUS.—Pendulum-balls of different weights. Fine wire for suspension. A suitable clamp for holding the wire.* An electric bell, or telegraph sounder, which is to be operated by a pendulum provided with a circuit-closing device. A bar-pendulum whose center of suspension may be shifted along the bar.

PRELIMINARY DISCUSSION.—This exercise should demonstrate the truth of the following principles:

(A) The law of isochronism.

---

* Two forms of pendulum clamps are shown in Fig. 38. The smaller one should be made of brass or strap-iron. The larger one, when made of hardwood, is very satisfactory.

(B) The period of a pendulum is independent of the weight of the pendulum.

(C) The law of length and time.

(D) The period of a compound pendulum having a given apparent length is different from that of a simple pendulum having the same length.

(E) A compound pendulum is equivalent to a given simple pendulum when the periods of the two are the same.

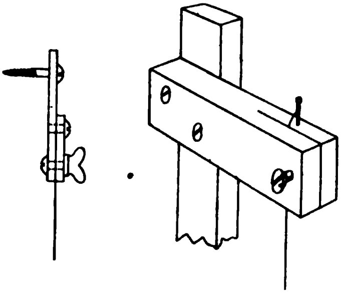
FIG. 38.

DIRECTIONS. (NOTE.—In all operations whose purpose is to determine the period of a pendulum, one student should count the time while another counts the number of vibrations. In each case set the pendulum swinging and begin the count at a prearranged signal.)

PART A.—(1) Set the pendulum swinging through an arc of about 10 cm. Count the number of vibrations made in, say, 3 minutes. Record the exact time and the number of vibrations.

(2) Set the pendulum swinging through an arc of about 20 cm. Proceed as directed in (1). Be very certain that the length of the pendulum has not been changed.

PART B.—(1) Using a pendulum-ball whose weight is about 100 grams, determine the number of vibrations which this pendulum makes in 3 minutes. Record the number of vibrations and the exact time. Carefully measure the length of this pendulum.

(2) Use a pendulum-ball whose weight is about 500 grams. Adjust the pendulum so that its length shall equal the length of the one just used. Proceed precisely as directed in (1).

(3) Suspend two pendulums side by side, one having a light ball and the other a heavy one. Adjust them so that their lengths shall be as nearly the same as you can make them. Set them both swinging together. Count the number of vibrations which each pendulum makes in 3 minutes.

Part C.—(1) Adjust a pendulum so that its length shall be about 50 cm. Count the number of vibrations made by it in 3 minutes. This pendulum may be assumed to approximate a simple one if the ball is comparatively heavy and the suspension wire is very light. Measure the length of this pendulum, l.

(2) Adjust the length of the pendulum just used so that its length shall be about 100 cm. Proceed as directed in (1), just preceding, and call the length of this pendulum l_1.

(3) Repeat (1) and (2).

Parts D and E.—(1) Adjust the bar-pendulum furnished you so that its length shall apparently equal that of the simple pendulum used in (2) of Part C. Determine the period of each.

(2) Adjust the length of the simple pendulum so that the periods of the two shall be the same. Measure the length of the simple pendulum.

Record data in a tabular form.

Calculations.—Compute for both lengths the period of the pendulum used in Part C. Substitute in the law of length and time, and verify.

Queries.—With reasonable care the student will be able to secure results which are accurate to a surprising degree. How may this be explained? What would follow if the law of isochronism were not true? Does the weight of the pendulum, or that of the ball, have anything to do with the period of the pendulum? What is the length of the compound pendulum used in Parts D and E?

EXERCISE 28

Determination of the Local Value of g

APPARATUS.—Same as that used in Exercise 27.

PRELIMINARY DISCUSSION.—The principal object of this exercise is to direct the attention of the student to the fact that the force of gravitation at a given place on the earth's surface depends upon the distance of the place from the center of the earth. This distance is determined by two things: (1) the altitude above sea, and (2) the latitude of the place. The effect of only the former will be considered in this exercise.

Since it is the force of gravity which causes the pendulum to vibrate, the rate of the vibration must be a measure of the force which produces it. Hence the period of a pendulum at a given place depends upon the value of the force of gravity at that place.

Now, the mutual relation which exists among the elements involved in a gravity pendulum is expressed by the equation,

$$g = \frac{\pi^2 l}{t^2}.$$

The student has no doubt already discovered that he is able to determine the period of a pendulum with far greater accuracy than he is able to measure its length. Hence it follows that, granting the truth of the law of length and time, the length of a pendulum, *as computed from its period*, will be found to yield more satisfactory results than its length as determined by direct measurement. With these ideas in mind, the student is asked to verify the above equation in two ways:

(A) By using a *directly measured* value of l.

(B) By using a value of l, *computed* from the period of the pendulum.

Directions.—(1) Measure the length of the pendulum with great care. Repeat this operation several times.

(2) Determine the period of the pendulum used. Repeat this operation at least three times.

Record data in a neat tabular form.

Calculations.—Using the directly measured length of the pendulum, substitute in the above equation, and verify.

Compute the length of the pendulum from its observed period, using the law of length and time.

Using this computed length, substitute and verify again.

Compare these results with each other and with the accepted local value of gravity.

CHAPTER IV

MECHANICS OF FLUIDS

EXERCISE 29

Cohesion of Water: Adhesion between Glass and Mercury, and between Iron and Mercury

Apparatus.—Rectangular pieces of glass and iron, about 7.5 cm. square, to be suspended from a balance, as shown in Fig. 39.

Directions.—(1) Before beginning the experiment, see that the plates are perfectly clean. The iron plate should be free from oil and rust. Place a block about a centimeter below the right-hand scale-pan to serve as a stop.

(2) Suspend the glass plate from the left-hand arm of the balance. See that the plate is suspended horizontally before proceeding further.

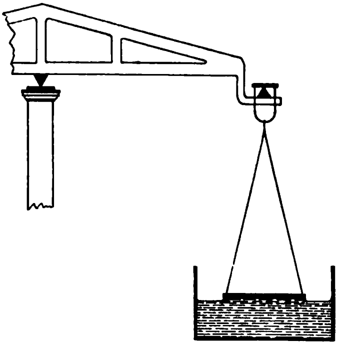

Fig. 39.

(3) Place a shallow glass vessel below the plate, as shown in the figure. Pour water into the vessel until the water rises and comes in contact with the under side of the plate. Be very careful not to get any water on the upper side of the plate.

(4) Now, *slowly* and *gently*, pour fine shot into the proper scale-pan of the balance until the glass plate is pulled away from the water.

(5) Weigh the shot in grams.

(6) Measure the length and width of the glass plate in centimeters.

(7) Repeat (1), (2), (3), (4), and (5), using mercury instead of water.

(8) Repeat (1), (2), (3), (4), and (5), using mercury and an iron plate.*

(9) Repeat (8).

Record data in the following form:

Liquid.	Area of Glass Plate in Sq. Cm.						
	TOTAL WEIGHT IN PAN.						SEPARATING FORCE IN GRAMS PER SQ. CM.
	Trial.						
	1	2	3	4	5	Mean.	
Water....							....
Mercury..							....

	Area of Iron Plate in Sq. Cm.						
	TOTAL WEIGHT IN PAN.						SEPARATING FORCE IN GRAMS PER SQ. CM.
	Trial.						
	1	2	3	4	5	Mean.	
Mercury..							....

Calculations and Results.—Compute the mean total weights called for in the table. Calculate the area of both plates and the separating force, in grams per sq. cm., for each case. *Each separating force is a measure of what?*

---

* An iron plate is suggested for the reason that it is less difficult to suspend than the glass plate. Its use also lends some variety to the exercise.

Queries.—Why is it so hard to lift a board straight up out of the water—i. e., to lift it out of the water, keeping it horizontal all the while? Why is it far easier to lift it out edgewise?

EXERCISE 30

Boyle's Law

Apparatus.—A Boyle's Law apparatus as shown in Fig. 40.* This form of the instrument may be readily and cheaply improvised by mounting a glass tube at each side of an ordinary meter-stick. The left-hand, or *A* tube, should be about 45 cm. long, and closed at its upper end. The *B* tube should be over a meter long. Both tubes should be joined at their lower ends to a Y-tube provided with a stop-cock. A barometer.

Preliminary Discussion.—In the performance of this exercise it will be necessary to measure the volume of the gas—air in this case—and the pressure to which the gas is subjected. In the operations which follow, let *a* be the height of the mercury surface in tube *B*, let *b* be that of the mercury surface in tube *A*, and *C* be the upper end of the closed tube. Then, in each case, the volume of the confined air will be given by $C - b$. (While it is true that $C - b$ is really the *length* of the air column, nevertheless it may be taken to represent the volume. Why?)

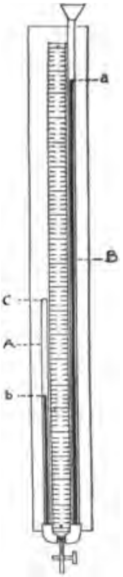

Fig. 40.

Part A.—**Proof of the law for pressures greater than one atmosphere.**

Directions.—(1) Preliminary to the operations of Part

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* The wording and method of this exercise may be easily adapted to other forms of apparatus—e. g., the sliding form, shown in Fig. 41.

A, the mercury in both tubes should be brought to the same level, about the middle of tube *A*.

(2) Pour mercury into tube *B* until the difference between the heights of the columns is about 10 cm. Take readings of *a*, *b*, and *C*.

(3) Proceed to make at least nine other pairs of observations—one reading of *C* being sufficient. The difference of level should be increased about 10 cm. each time. Record readings *a* and *b* for each case.

(4) Repeat the operations of (2) and (3), but in the reverse order. This may be done, and a series of *decreasing* pressures may be obtained, by opening the stop-cock and allowing some mercury to run out. Decrease the difference of level by about 10 cm. each time. It is not necessary to be exact in this particular. Record readings *a* and *b* for each case.

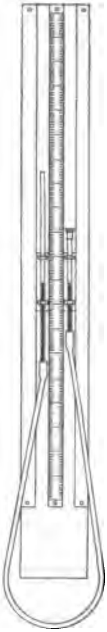

FIG. 41.

PART B.—**Proof of the law for pressures less than one atmosphere.**

PRELIMINARY DISCUSSION.—If, when the levels of the mercury are the same, the stop-cock be opened, the mercury which runs out will cause the column in tube *B* to stand at a lower level than that of the column in tube *A*. Now, it is evident that this will indicate an excess of mercury pressure in the closed tube. Hence it follows that the pressure of the confined air, *plus* the downward pressure of the mercury in the closed tube, is exactly equal to the atmospheric pressure acting upon the mercury surface in tube *B*. Whence it appears that whenever the column in *B* stands below that in *A*, the pressure of the confined air *is less than one atmosphere*.

DIRECTIONS.—(1) Observe the same preliminary direction that is given in Part A.

(2) By allowing mercury to run out, cause the column

in *B* to stand about 10 cm. below the column in *A*. Record the readings *a* and *b*.

(3) In the same manner proceed to make several other observations. Cause the column in *B* to stand about 10 cm. lower each time. (Caution: In no case should the mercury be allowed to fall below the bend.) Record the readings *a* and *b* for each case.

(4) Repeat (2) and (3), but in the reverse order. This may be done by pouring mercury into *B*. Increase the height of the column in *B* by about 10 cm. each time. Record readings *a* and *b* for each case.

Record data in the following form:

PART.	Trial.	*a*.	*b*.	Height of mercury col. in B. (*a* – *b*.)	Total press. in cm. of mercury.	Vol. of confined air. (*C* – *b*.)
(A)	1 2 3 4 5 Etc.					

PART.	Trial.	*a*.	*b*.	Height of mercury col. in A. (*b* – *a*.)	Total press. in cm. of mercury.	Vol. of confined air. (*C* – *b*.)
(B)	1 2 3 4 5 Etc.					

Height of barometer ... . cm.

CALCULATIONS AND RESULTS.—First, under this head in your report, state Boyle's Law in the form of a proportion. It is this proportion which the work of this exercise is intended to verify.

Compute the total pressure for each case. In Part A the total pressure is the mercury pressure as recorded in column 5 of the table, *plus* the atmospheric pressure for the day and hour as indicated by the barometer. Comparative results are sufficient in this exercise. Therefore, the total pressure may be conveniently expressed in centimeters of mercury. In Part A, the total pressure in each case will then be represented by the height of the barometer, *plus* the height of the mercury column, $a - b$. In Part B, the total pressure may be represented by the height of the barometer, *minus* the height of the mercury column, $b - a$. In every case the volume of the confined air may be represented by $C - b$.

Compute the total pressure and volume for each case, as just directed.

Substitute pairs of related values in the proportion you have written, and verify.

Using total pressure heights as abscissas, and volumes as ordinates, plot a curve showing the relation between them.*

Problem.—The gases used for calcium light are now pumped into steel cylinders under high pressure. In ordinary language we may say that a large amount of gas is thus caused to occupy a small amount of space. Suppose one of these cylinders to have a volume of 4 cubic feet. If the gas which it contains is under a pressure of 225 pounds per square inch, what will be the volume of the gas if it is allowed to escape into the atmosphere on a day when the barometer stands at 29 inches?

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* Suggestion.—Take an arbitrary set of values of P and V, assuming Boyle's Law to be rigorously correct for all pressures. Plot a curve with these values. Determine the character of this curve and look up its name, if you do not know it already. Compare this curve with the one drawn as directed above. See Hastings and Beach, p. 223.

EXERCISE 31

Principle of Archimedes

Apparatus.—A beam balance. A solid metal cylinder. A beaker of suitable capacity. Distilled water. Micrometer or vernier calipers.

Proposition.—It is required to prove that a body submerged in a fluid apparently loses in weight an amount equal to the weight of the fluid displaced; or, differently expressed, that a body is buoyed up by a force equal in amount to the weight of the fluid displaced. The apparent loss in weight, or the buoyancy, W, is given by the equation,

$$W = W_1 - W_2,$$

where W_1 is the weight of the body in air and W_2 is its weight in the given fluid.

Directions.—(1) Attach the cylinder by a thread or a fine wire to the left-hand arm of the balance and weigh it, first in air and then while submerged in water. Call the first weight W_1 and the second one W_2.

(2) Measure the length and diameter of the cylinder with either of the calipers.

(3) Repeat (1) and (2) several times.

Calculations and Results.—Compute the volume of the cylinder. Then compute the weight of that volume of water. This is the weight of the water displaced by the cylinder.

Substitute in the above equation and solve for W.

Compare the two values of W.

Queries.—Which value of W should be more nearly correct? Why?

Problems.—An ocean liner's gross weight is 8,000 tons. What is the weight of the water she displaces? What is meant by the "displacement" of a vessel? A water-logged block of wood, 1 × 1 × 1 foot, sinks evenly in pure, fresh water until the upper surface of the block is just on a level with the surface of the water. How much does the block weigh?

EXERCISE 32

Specific Gravity of Solids Denser than, and Insoluble in, Water

APPARATUS.—A beam balance. Pieces of brass, iron, and lead. Distilled water.

DIRECTIONS.—(1) Weigh the brass, iron, and lead in air separately.

(2) Weigh each in water, separately, suspended from the left-hand arm of the balance.

(3) Repeat (1) and (2) several times.

CALCULATIONS AND RESULTS.—When A is the weight in air and B is the weight in water, the specific gravity of a dense solid is given by the equation,

$$\text{Specific Gravity} = \frac{A}{A - B}.$$

Compute the specific gravity of each substance used.

Compare your results with the values given in tables in the Appendix.

QUERIES.—What is standard water? Is the specific gravity of the above substances, as determined by you, too high or too low? Give reasons for your answer. What principle is used in this exercise? To be strictly correct, should the substance be weighed in air? If not, in what should it be weighed? What is the quantity $A - B$? How could you find the specific gravity of a substance, *soluble in water*, by the above method?

EXERCISE 33

Specific Gravity of Solids Less Dense than, and Insoluble in, Water

Apparatus.—A beam balance. A sinker of convenient size. Distilled water. Pieces of cork, wax, and other light substances.

Directions.—(1) Weigh the cork in the left-hand pan of the balance, calling this weight A, as in Exercise 32.

(2) Remove the cork from the pan and attach the sinker to the left-hand arm of the balance by means of a thread or a fine wire. Weigh the sinker while it is submerged in water. Call this weight S.

(3) Now, attach the sinker to the cork, and weigh both while submerged in water. Call this weight S_1.

(4) Repeat (1), (2), and (3), using other light substances.

Calculations and Results.—From the three weights, A, S, and S_1, calculate the specific gravity of each substance.

Deduce a formula for the determination of the specific gravity of light substances by the method used in this exercise. Explain the derivation of this formula.

EXERCISE 34

Specific Gravity of a Liquid by the Method of weighing a Solid in it*

Apparatus.—A beam balance. A set of weights. At least two liquids of different specific gravity.

Directions.—(1) Weigh some small object—e. g., a glass stopper, in air. Call this weight W_1.

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* It will lend additional interest to the work to use the same liquids in this and in the three succeeding exercises.

(2) Weigh the same object in distilled water, suspended from the left-hand arm of a good balance. Call this weight W_2.

(3) Weigh the same object in the liquid whose specific gravity is to be determined, suspended as directed in (2). Call this weight W_3.

(4) Repeat (1), (2), and (3) several times.

(5) Repeat (1), (2), and (3), using some other liquid whose specific gravity is unknown to you.

Record data in a neat form.

Calculations and Results.—In any determination of specific gravity two quantities are always involved:

(1) The weight of a certain, but not necessarily known, volume of the substance whose specific gravity is sought.

(2) The weight of an *equal* volume of standard water.

From the three weights, W_1, W_2, and W_3, these two quantities can be determined. Determine these quantities, and then compute the specific gravity of each liquid furnished you. (Use mean values of W_1, W_2, and W_3.)

Deduce a formula for the computation of specific gravity by the method of this exercise. Give this formula in your report, and show how it was deduced.

EXERCISE 35

Specific Gravity of a Liquid by Means of a Specific-Gravity Bottle

Apparatus.—A beam balance. Distilled water. A thermometer and a shallow vessel in which water is to be kept at a temperature slightly above that of the room.

Directions.—(1) Clean and dry the bottle thoroughly. Weigh it with the cork lying beside it in the pan of the balance.

(2) Fill the bottle with a liquid whose specific gravity is to be found. Fill it to the top of the neck; then put the

ground stopper in place, slowly and with great care, inclining the bottle slightly so as to allow air-bubbles to escape. Now, place the bottle in the shallow pan and let it remain there until the temperature of the liquid in the bottle has come to that of the water in the shallow pan. As soon as the temperatures have equalized, take the temperature of the water in the pan. Remove the bottle from the pan and wipe the bottle perfectly dry. (Is there any need for haste in wiping the bottle preparatory to weighing it?) Weigh the bottle with its contents.

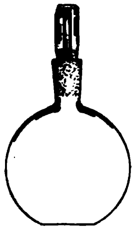
FIG. 42.

(3) Empty the bottle. If it has contained a liquid which is *insoluble* in water, the bottle must be thoroughly cleaned and dried. If, on the contrary, it has contained a liquid *soluble* in water it will be sufficient to rinse it well with distilled water. Fill the bottle with distilled water. Place it in the shallow pan and bring the temperature of the water in the bottle and in the pan to the temperature noted as directed in (2). Insert the cork as directed above. Dry the bottle and then weigh it.

(4) Proceed as directed in (1), (2), and (3), using another liquid whose specific gravity is unknown to you.

Record data in a tabular form.

CALCULATIONS AND RESULTS.—Compute the specific gravity of each liquid. Compare results with the values given in tables in the Appendix.

QUERIES.—What is the object of having the temperature of the liquid in the bottle the same in operations (2) and (3)? What can you say of the accuracy of this method as compared with that of the preceding exercises on specific gravity?

EXERCISE 36

Specific Gravity of Liquids by Hare's Method

Apparatus.—A Hare's apparatus, consisting essentially of an inverted Y-tube, each lower arm of which dips into a small beaker. One beaker is to be filled with distilled water and the other with a liquid whose specific gravity is to be found.

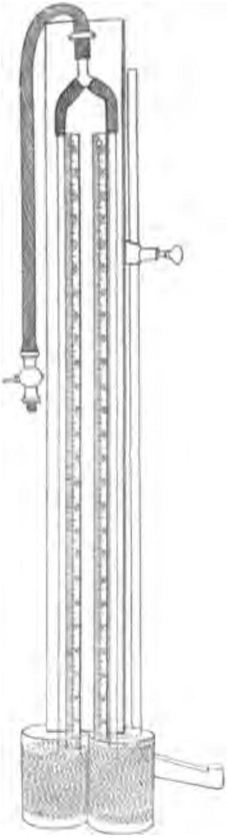

Fig. 48.

Directions.—(1) By suction remove enough air from the tubes to cause the distilled water column to stand about half-way to the top of the tube. Close the stop-cock and observe whether the columns remain stationary. If they do not, there is a leak somewhere which must be closed. Take and record the height of both columns, calling that of the distilled water column *H*, and that of the other column H_1.

(2) Remove air from the tubes until the lighter liquid stands nearly at the top of the tube. Take and record the height of both columns.

Caution.—Before removing either beaker, open the stop-cock.

(3) Repeat (1) and (2).

(4) Repeat (1), (2), and (3), using another liquid whose specific gravity is unknown to you.

(5) Repeat (4).

CALCULATIONS AND RESULTS.—By Hare's method, the specific gravity of a liquid is given by the equation,

$$\text{Specific Gravity} = \frac{H}{H_1},$$

where H is the height of the distilled water column and H_1 is the height of the other column.

Substitute in the foregoing equation and solve for the specific gravity of each liquid used.

PROBLEM.—Draw a diagram to represent the apparatus used in this exercise. Letter it, showing the height of both columns. Assume numerical values for these heights and *prove* that the above equation is correct.

QUERY.—In your judgment, how does the accuracy of this method compare with that of the methods already employed? Give reasons for your opinion.

EXERCISE 37

Specific Gravity of Liquids: By Means of a Densimeter

APPARATUS. — Two hydrometer jars. Two wooden densimeters, one for liquids heavier than water, the other for those lighter than water.

DIRECTIONS.

PART A.—**To find the specific gravity of a liquid denser than water.**

(1) Fill a clean hydrometer jar with distilled water and place the proper densimeter in it. Take the reading of the water displaced, looking underneath the surface of the water, as shown in Fig. 14. Call this reading D.

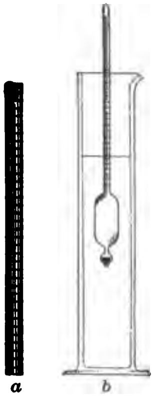

FIG. 45.

(2) Remove the densimeter from the distilled water. Wipe it perfectly dry and introduce it

into a jar filled nearly full of liquid heavier than water. Take the reading as directed in (1), calling it D_1.

PART B.—**To find the specific gravity of a liquid less dense than water.**

(1) Proceed as directed in (1) and (2) of A, using the proper densimeter and a liquid lighter than water, calling the displacement in water D, and that in the lighter liquid D_1, as before.

Record data in a neat form.

CALCULATIONS AND RESULTS.—The specific gravity of any liquid determined by the method used in this exercise is given by the equation,

$$\text{Specific Gravity} = \frac{D}{D_1},$$

where D is the displacement of standard water and D_1 is that of the given liquid. Compute the specific gravity of each liquid experimented with. *Prove the correctness of the above equation.*

QUERY.—Could you graduate the stems of the densimeters you have used so as to make them direct-reading—that is, to give specific gravities without computation?

NOTE.—A direct-reading Beaumé hydrometer is shown at *b* in Fig. 44. Describe the usual method of graduating these hydrometers. See Deschanel's Physics, Part I, p. 113.

CHAPTER V

HEAT

EXERCISE 38

Boiling and Freezing Points on a Thermometer *

Apparatus.—A centigrade thermometer graduated from – 10° to 100°. A vessel having a capacity of about a liter. A simple form of hypsometer, as shown in Fig. 45. (A very satisfactory and durable hypsometer can be made by the local tinner at small expense.)

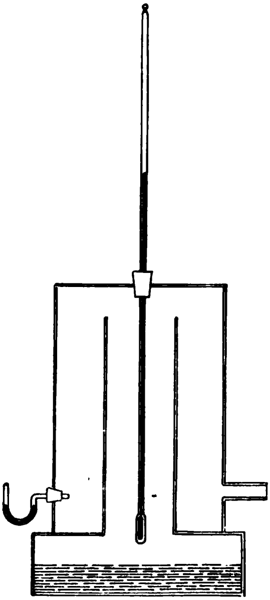

Fig. 45.

Propositions. — (1) The fixed points on a thermometer are based upon the freezing and boiling points of pure water under a pressure of one atmosphere.

(2) The freezing and boiling points of any liquid depend upon the pressure to which the liquid is subjected.

(3) Different liquids have different freezing and boiling points under the same conditions.

Directions. — (1) Pack fine ice chips around the thermometer up to the freezing-point. Leave the thermometer thus for several minutes. Then, by carefully watching the mercury column, determine whether the column has become stationary. When the mercury becomes stationary, such fact indicates that the freezing-point has been reached. Read and record the temperature of the melting ice, estimating the fractional degree in tenths. Avoid the error of parallax in making this reading.

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* Deschanel, Heat, p. 4; Whiting, pp. 109, 140.

(2) Place the thermometer in the hypsometer, after the latter has warmed to the temperature of the room. Be certain that the bulb of the thermometer is at least an inch above the water in the hypsometer. Then heat the water to the boiling-point. When the mercury has become stationary, read and record the observed boiling-point. Record the height of the barometer in centimeters.

(3) *Under the supervision of the instructor*, partially close up the tube or opening through which the steam escapes from the hypsometer.* What do you know about the pressure on the boiling water, compared with what it was a moment ago? Observe the effect of this change of pressure on the boiling-point, recording the highest point reached. Open the vent and observe what happens. If it is thought desirable by the instructor, this part of the exercise may be extended as follows: Fill an 8-ounce flask about half full of water. Pass a thermometer through a two-hole rubber stopper and insert the stopper in the flask. Bring the water in the flask to the boiling-point. Quickly close the other hole in the stopper and invert the flask over a large basin. Pour cold water on the flask. The water will continue to *boil* for some time. What do you now know about the pressure on the boiling water compared with what it was in the first part of this operation? When the water has almost ceased to boil, observe the temperature, recording this as the lowest boiling-point reached in the experiment.

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* Great care must be exercised in doing this, or an explosion may result.

(4) Heat some water in the liter vessel to about 90° C. *Turn out the Bunsen flame.* Then fill a test-tube half full of 90-per-cent alcohol. Place the test-tube in the warm water. Stir the alcohol with a thermometer and take its temperature at the instant it is on the point of ceasing to boil. Record this as the boiling-point of the alcohol furnished you. If other liquids having low boiling-points are at hand, determine their boiling-points in the same way.

Record all data in a tabular form.

RESULTS.—It has been found that pressure affects the freezing-point but very little. On the contrary, change of pressure influences the boiling-point to a marked degree. Now, the effect of a change of pressure is such that we say the boiling-point *varies with* the pressure, but is *not proportional* to the pressure. When the pressure is *increased* by an amount equivalent to that of 26.8 mm. of mercury, the boiling-point is *raised* 1° C. Conversely, when the pressure is *diminished* by the same amount, the boiling-point is *lowered* 1° C.

The boiling-point of pure water under a pressure of 760 mm. of mercury is 100° C. Compute the boiling-point of pure water under the normal atmospheric pressure for the locality of your laboratory. Compare this computed boiling-point with the boiling-point observed on your thermometer after the latter has been reduced to normal local pressure.

Your report should clearly show how the operations of this exercise verify the above propositions.

QUERIES.—When we say that the temperature of a certain body is 30° C. or Fahr., what do we mean? When we say that the temperature of a body has been raised 30°, what does the number 30 represent? The measurement of temperature by thermometers is based upon the assumption that the expansion of the thermometric substance is sensibly uniform: why is this assumption of vital importance in thermometry?

EXERCISE 39

Linear Expansion Coefficient for Brass and Iron

Apparatus.—A zinc or galvanized-iron tube fitted at both ends with water- and steam-tight corks, through which extends a brass or iron rod. One end of the rod rests against an arm which is usually attached to one of the uprights which support the tube. The other upright carries a micrometer, which may be brought in contact with the other end of the rod. The tube is provided with inlet and outlet tubes, by means of which water or steam may be caused to pass through the main tube. A telegraph sounder, or a galvanoscope, and one or two voltaic cells are connected so as to be in circuit with the rod and the micrometer screw. The contact of the screw with the rod closes the circuit, which is indicated by the operation of the sounder or galvanoscope.

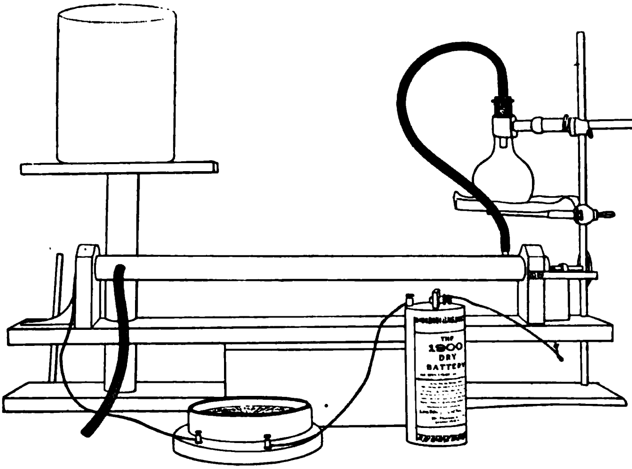

Fig. 46.

Preliminary Discussion.—Within comparatively small ranges of temperature, expansion is quite uniform. Therefore, a practical use of such coefficients can be made.

The expansion coefficient of any substance *is always a fraction.* The linear-expansion coefficient is that fraction of the length of the body at a specified temperature—usually 0° C.—by which its length is increased or decreased when the temperature is changed 1° C.

Suppose, for example, that the linear-expansion coefficient of a certain substance is 0.000012. This statement means that if the temperature of the body is changed 1° C., the length of the body will be changed by 0.000012 of its length at 0° C. Suppose its length at 0° C. to be 12 feet, and its temperature to be increased 100° C., what will be the length of the body at 100° C.?

By definition the increase in length is equal to

$$12 \text{ feet} \times 0.000012 \times 100.$$

Whence final length = 12.0144 feet.

Directions.—(1) See that one end of the rod rests securely against the end-piece. Turn the micrometer screw back far enough to allow for the expansion of the rod. Connect the tube to a steam-generator of some sort, and allow the steam to pass through the tube for several minutes. Record the height of the barometer. This is to be used to compute the higher temperature of the rod.* While the steam is still passing, turn the micrometer screw until it makes contact with the rod. Take the linear and circular scale-readings of the micrometer screw with great care.

(2) Disconnect the boiler from the tube. Again see that the rod, when cool, rests against the end-piece. Now cause ice-water to flow from an elevated vessel through the tube for several minutes. Then turn up the micrometer screw until it makes contact with the rod, and take both readings of the micrometer screw, as before. While this is

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* Compute it as directed in Exercise 38.

being done, let a fellow student take the temperature of the water both before and after entering the tube. Call the mean of these temperatures the lower, or zero, temperature of the rod.

(3) Measure the length of the rod while it is at, or near, 0° C.

RESULTS.—(1) Expansion coefficients are usually represented by the letter a. From the micrometer readings compute the *total* expansion of the rod. Then compute a, the expansion *per unit of length*, *per degree* centigrade.

Compare your result with the value of this coefficient given in the Appendix.

(2) From the definition of the linear-expansion coefficient and from the solution of the problem given above, deduce an equation for computing a linear-expansion coefficient.

Substitute numerical values in the equation you have framed, and note that the performance of the indicated operation is practically identical with the calculation made as directed in (1).

PROBLEM.—Suppose a certain continuous steel tube to be 2,000 feet long at 0° C. What will be its length at 53° C.?

EXERCISE 40

Expansion Coefficient of a Liquid

APPARATUS.—Two glass tubes arranged as shown in Fig. 47. The outer one should be about 110 cm. long. The inner tube should be large enough to allow the micrometer screw to turn freely within it. A thermometer. A steam-generator.

PRELIMINARY DISCUSSION.—The expansion coefficient of a liquid is a *volume coefficient*. It is that fraction of the volume at a selected temperature by which the volume is increased or decreased when the temperature of the liquid is changed 1° C. For example, suppose that a certain liquid has a volume of 50 cc. at 0° C., and that its mean expansion coefficient between 0° and 100° C. is 0.0005. What will be the volume of the liquid at 20° C.?

By definition, the increase in volume will be

5 cc. × 0.0005 × 20.

Whence, final volume = 50.5 cc.

In computing a volume coefficient, the original and final volumes, as well as the change of temperature, must be known. The volume of a cylinder is proportional to its length. Hence, the original volume may be represented by the length, since the body of liquid under consideration is cylindrical. For the same reason, the increase of volume may be represented by the increase in length.

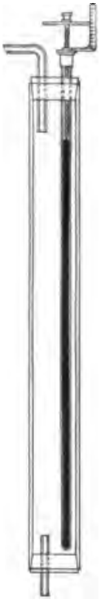

FIG. 47.

DIRECTIONS.—(1) Nearly fill the inner tube with a liquid whose expansion coefficient is to be found. (Fill the tube to a point about 12 cm. below the lower end of the screw when it is at its highest position.) Connect the outer tube to an elevated vessel, and pass ice-water through the outer tube several minutes. At the end of this time, and while the ice-water is still flowing through the tube, take the reading of the micrometer screw. Before taking these readings, be certain that the lower end of the screw just touches the surface of the liquid. Take the temperature of the water both before and after it passes through the tube. Measure the length of the liquid column in the inner tube.

(2) Arrange the apparatus to pass steam through the outer tube. Pass steam through the tube for about 10 minutes. Take the micrometer readings as directed in (1). Read and record the height of the barometer.

(3) Repeat (1) and (2).

Calculations and Results.—From the barometer reading compute the temperature of steam for the day. From the micrometer readings compute the total expansion for each trial. Then compute *a*, the expansion per unit of volume, per degree centigrade. Compare your result with the value of this coefficient given in the Appendix.

From the definition of a volume coefficient and from the solution of the problem given above, deduce an equation for computing a volume-expansion coefficient.

Substitute numerical values in the equation you have framed, and make the observation noted in the latter part of (2) of calculations in the preceding exercise.

EXERCISE 41

Expansion Coefficient of Volume of Gases under Constant Pressure

Apparatus.—A glass tube, about 45 or 50 cm. long and of 1 mm. bore, sealed at one end, and bent at right angles about 10 cm. from the open end. A glass tube of suitable length and diameter, closed with two-hole rubber corks. A vessel of such length that the glass tube may be placed in it in a horizontal position. Near the bend are placed two movable markers, *a* and *b*.

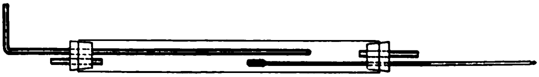

Fig. 48.

Preliminary Discussion.—When the temperature of a gas is raised 1° C., the volume of the gas is increased by about 0.00367 of its volume at 0° C. In other words, the expansion coefficient of volume of gases is this fraction. The purpose of this exercise is twofold:

(1) To prove, approximately, that the expansion coefficient of volume of gases under constant pressure is 0.00367.

(2) To prove the Law of Charles,

$$\frac{V}{V_1}=\frac{T}{T_1},$$

where V and T are one volume and absolute temperature, respectively, of a certain body of gas, and V_1 and T_1 are another volume and absolute temperature, respectively, of the same body of gas—the pressure remaining the same throughout the change of volume and temperature.

DIRECTIONS.—(1) Introduce a short filament of mercury into the bent tube. To do this, warm the tube to expel some of the air, and then, before the air in the tube has had time to cool, insert the open end into a vessel of mercury. On removing the tube at the proper moment, a filament of the proper length, about 5 mm., will be drawn into the tube. If this filament does not come to rest about 10 cm. beyond the marker, in the lower arm of the tube, this operation must be repeated until it does so.

(2) Place the tube in a horizontal position in a bath of ice-water. Locate a marker so that its inner edge shall coincide with the inner end of the mercury filament when the latter has come to rest.* Take the temperature of the ice-water.

(3) Now, without disturbing the marker just located, arrange the apparatus as shown in the figure. Make the proper connections and pass steam through the larger tube for several minutes. When you are sure that the air is heated to the temperature of the steam, adjust the other, or *b* marker, as directed in (2), to indicate the position which the inner end of the filament takes under the action of the expanding air. Read and record the height of the barometer for the day and hour.

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* By the inner marker or the inner end of the mercury filament is meant the marker or end nearer the closed end of the bent tube.

(4) Remove the tube carefully and place it on the scale of the meter-stick. Determine the distance between the inner edges of the markers. This length will represent the *increase of volume.* Also, measure the distance from the closed end of the tube to the inner edge of the marker *a.* (Make an allowance for the curvature of the tube.) This length will represent the *original volume* of the gas.

(5) Repeat (1), (2), (3), and (4).

CALCULATIONS AND RESULTS.—From the reading of the barometer, compute the temperature of steam for the day. Knowing the original and final volumes of the gas, and the change in temperature, compute the coefficient of expansion. Compare your result with the value of this coefficient given in the Appendix.

Deduce an equation for computing an expansion coefficient of volume of gases under constant pressure.

From the temperatures observed as directed in (2) and (3), compute the corresponding absolute temperatures, T and T_1, of the equation given in the preliminary discussion. Substitute numerical values in this equation, and verify.

EXERCISE 42

Expansion Coefficient of Pressure in Gases at Constant Volume

APPARATUS.—A simple form of Regnault's apparatus, consisting essentially of a flask suspended within a closed vessel and connected to a U-tube of glass. The U-tube should be partially filled with mercury. The flask should contain dry air, introduced by the instructor, or an advanced student, prior to the beginning of the experiment. The U-tube should be mounted on a millimeter scale.

PRELIMINARY DISCUSSION.—In Exercise 41 the attention of the student was directed to the fact that the volume coefficient of expansion of gases is 0.00367. But if the gas is heated *while its volume is kept constant*, it is evident that the pressure of the gas must increase. The purpose of this exercise, therefore, is threefold:

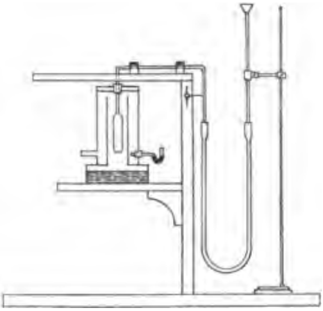

FIG. 49.

(1) To determine, approximately, the coefficient of increase of pressure in gases at constant volume.

(2) To show that the coefficients of volume and pressure are practically the same, numerically.

(3) To prove that

$$\frac{P}{P_1} = \frac{T}{T_1},$$

where P and T are one pressure and absolute temperature, respectively, of a given body of gas, and P_1 and T_1 are another pressure and absolute temperature, respectively, of the same body of gas, the volume remaining the same throughout the change of pressure and temperature.

DIRECTIONS.—(1) Pass ice-water through the vessel for several minutes. In performing this operation the zero point on the thermometer should be slightly above the lid of the vessel. When the temperature of the air in the flask has become stationary, read the height of the mercury column nearer the closed vessel. *This point must be carefully noted.* See (2). Then read the height of the outer mercury column. The difference between the heights of the columns will give the mercury pressure acting upon the air confined in the flask. This pressure, plus that of the atmosphere, will be the total pressure acting upon the confined air. To get the atmospheric portion of the total pressure, read the barometer. Take the temperature of the ice-water as directed in (2) of Exercise 39.

(2) Now, pass steam through the closed vessel until the temperature of the air in the flask becomes stationary.

As soon as the temperature of the air begins to rise, the mercury in the outer tube will also rise. Pour some mercury into the outer tube from time to time. If the apparatus is not watched closely the expanding air will drive all the mercury out of the outer tube. Endeavor to keep the level of the inner column at the height observed when ice-water was passing. As soon as the temperature becomes stationary, adjust the height of the outer column until the level of the inner column is precisely what it was when the ice-water was passing. If this operation is carefully performed, the condition of a constant volume will be nearly fulfilled. Read and record the height of both mercury columns with great care. Read the barometer.

(3) Repeat (1) and (2) if there is time.

Record data in tabular form.

CALCULATIONS AND RESULTS.—From the reading of the barometer compute the temperature of steam for the day. Compute the total pressure acting upon the confined gas when the ice-water was passing. Express this pressure in centimeters of mercury. Call it P_0. Compute the total pressure acting upon the gas when the steam was passing. Express this pressure also in centimeters and call it P_1. Then the coefficient of increase of pressure may be computed by means of the equation,

$$a = \frac{P_1 - P_0}{P_0 t},$$

where a is the coefficient of pressure and t is the change of temperature. Compare this result with the accepted value of the coefficient of pressure and with the coefficient of increase of volume.

From the temperature of the ice-water and of steam compute the corresponding absolute temperatures. Substitute numerical values in the equation given in the preliminary discussion, and verify.

EXERCISE 43

Melting-Point of Paraffin

APPARATUS.—A short piece of glass tubing, of about 1 mm. bore, closed at one end and partially filled with paraffin. A thermometer. A 6-ounce beaker.

DIRECTIONS.—(1) Place the tube containing the paraffin in a beaker nearly full of water. Hold the thermometer close to the tube. Neither should be allowed to touch the beaker. If it is thought desirable, the thermometer may be tied to the tube containing the paraffin, and both suspended according to the direction just given.

(2) Slowly heat the water. Watch the paraffin, and when the beginning of melting is indicated by the paraffin becoming partially translucent, immediately take the reading of the thermometer.

(3) After the paraffin has completely melted, allow the water to cool. A few small ice chips dropped into the beaker will expedite the cooling. Watch for the first indication of opacity in the cooling paraffin. When this is observed, immediately take the reading of the thermometer.

(4) Repeat (1), (2), and (3) until fairly concordant results are obtained.

RESULTS.—Compare the melting-point of paraffin as determined by you with the value given in the Appendix. Quite a discrepancy is likely to exist between the two values. The student should not be discouraged by this, for the reason that the melting-points of such substances as paraffin are really difficult to determine with any considerable degree of accuracy. Furthermore, different specimens of such substances as the waxes and the heavy fats may have rather widely different melting-points.

EXERCISE 44

Calorimetry: Method of Mixtures

Apparatus.—A simple form of calorimeter consisting of two vessels of such size that one may be placed within the other so as to leave an air space of about 5 cm. between them on all sides. A wooden or cork lid should be fitted to the outer vessel so as to rest on the inner one. A thermometer may be introduced into the calorimeter through a hole in the lid. An extra pail. A Bunsen burner.

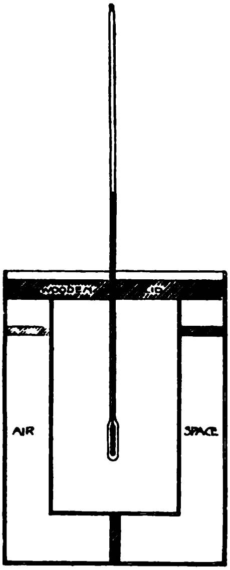

Fig. 50.

Preliminary Discussion.—Quantitative measurements of heat are usually made by the method of mixtures. This method is the basis of the work in heat which follows. It is the purpose of this exercise to determine the temperature of a mixture of *unequal* quantities of water which were at *different* temperatures before mixing.

Directions.—(1) See that the calorimeter is perfectly dry. Weigh it with the lid and the thermometer in place. Call this weight W_c. Put about 500 cc. of cold water in the calorimeter and weigh. Subtract the weight of the calorimeter and thermometer from the total weight. Record the weight of the remainder as the weight of the cold water, W_{cw}.

(2) Now heat about 700 cc. of water to about 40° C. Take the temperature, T_{cw}, of the cold water. Take the temperature, T_{hw}, of the hot water, and immediately pour the hot water into the calorimeter.

(3) Stir the mixture with the thermometer, and, as soon as the mercury becomes stationary, take the temperature, T_m, of the mixture. Then weigh the calorimeter with its contents to determine the weight, W_{hw}, of the hot water.

Record data in a tabular farm.

THEORY OF THE METHOD OF MIXTURES

It is evident that the calories which the hot water *lost* by mixing with the cold water must have been *gained* by the cold water. But if we assume that the calories lost by the hot water equal the calories gained by the cold water, we will thereby introduce a considerable error, for the reason that the calorimeter and the thermometer must also have their temperature raised to that of the mixture. The heat thus consumed will subtract from the total heat which otherwise would be available for raising the temperature of the cold water (neglecting the small amount lost by conduction and radiation). Now, the number of calories necessary to raise the temperature of the calorimeter to that of the mixture is equal to the weight of the calorimeter, multiplied by the rise of temperature, multiplied by the specific heat of the calorimeter.* Hence, the equation for the method of mixtures may be written thus:

$$\left.\begin{array}{l}\text{Calories lost by}\\ \quad\text{hot water}\end{array}\right\} = \left\{\begin{array}{l}\text{Calories gained by cold water, plus}\\ \quad\text{calories gained by calorimeter.}\end{array}\right.$$

Using letter values, this equation becomes,

$$W_{hw}(T_{hw} - T_m)\ (Sp.\,H_w) = \left\{\begin{array}{r}W_{cw}(T_m - T_{cw})\ (Sp.\,H_w) + \\ W_c(T_m - T_{cw})\ (Sp.\,H_c),\end{array}\right.$$

where $Sp.\ H_w$ is the specific heat of water, and $Sp.\ H_c$ is the specific heat of the substance of which the calorimeter is made.

CALCULATIONS.—Substitute in the equation and solve for T_m.

---

* To be more exact, the heat consumed by the thermometer should be added to this amount.

EXERCISE 45

Calorimetry: Specific Heat of Iron

Apparatus.—Same as that of the preceding exercise. A piece of iron whose specific heat is to be determined.

Preliminary Definition.—*Numerically*, the specific heat of any substance is the number of calories necessary to raise the temperature of 1 gram (or 1 kg.) of the substance 1° C.

Directions.—(1) Weigh the piece of iron. Record its weight, W_i. Suspend it in water at the boiling-point. Allow the iron to remain in the boiling water until it has acquired the temperature of the water. Take and record the temperature of the boiling water, assuming it to be the temperature of the iron, T_i.

(2) Dry the calorimeter. Weigh it. Record its weight, W_c. Put enough cold water into the calorimeter to completely cover the iron after it has been transferred to the calorimeter.

(3) Now weigh the calorimeter with the cold water in it. Subtract the weight of the calorimeter from the total weight, and record the remainder as the weight of the cold water, W_{cw}. Take and record the temperature of the cold water, T_{cw}.

(4) Transfer the iron quickly to the calorimeter. Stir the water in the calorimeter with the thermometer. As soon as the mercury has become stationary, take and record the temperature of the mixture, T_m. (Why call this the temperature of the mixture?)

Record data in a tabular form.

Calculations and Results.—Frame an equation, based upon the method of mixtures as developed in Exercise 44, for computing specific heats.

Substitute numerical values in this equation and solve for the specific heat of iron. Compare your result with the value given in the Appendix.

QUERIES.—As applied to every-day life, what is the meaning of specific heat? Suppose you were required to find the temperature of a piece of red-hot iron. Could you do so by the method used in this exercise? Explain how.

EXERCISE 46

Calorimetry: Heat of Fusion of Ice

APPARATUS.—Same as that for Exercise 44.

PRELIMINARY DISCUSSION.—The expression, the heat of fusion of ice, refers to a *numerical quantity.* This quantity is the number of calories necessary to melt 1 gram, or 1 kilogram, of ice at 0° C. *without raising its temperature.*

DIRECTIONS.—(1) Dry the calorimeter. Determine its weight, W_c. Put about 1 kg. of water at 60°, or thereabouts, into the calorimeter. Determine the weight of the water, W.

(2) Break up about 250 grams of ice into small pieces. Dry them. Take the temperature of the water, T_w, and immediately drop the ice-chips into the calorimeter. Just as the last bits of ice are on the point of melting, stir the mixture thoroughly, and as soon as the mercury becomes stationary, take the temperature of the mixture, T_m.

(3) Weigh the calorimeter with its contents. Compute the weight of the ice, W_i.

(4) Repeat (1), (2), and (3).

Record data in a tabular form.

CALCULATIONS AND RESULTS.—Evidently the ice was melted, and the ice-water at 0° C. resulting therefrom was raised to the temperature of the mixture. The heat necessary to do both was furnished by the water and the calorimeter. That is,

$$\left.\begin{array}{l}\text{Calories gained by ice} + \\ \quad\text{calories gained by ice-} \\ \quad\text{water}\end{array}\right\} = \left\{\begin{array}{l}\text{Calories lost by water} + \\ \quad\text{calories lost by calorim-} \\ \quad\text{eter.}\end{array}\right.$$

Rewrite the foregoing equation, using the letter values given in the directions. Then substitute numerical values in the equation, and solve for the heat of fusion of ice. Compare your result with the accepted value.

EXERCISE 47

Calorimetry: Heat of Vaporization of Water

Apparatus.—A calorimeter. A steam-generator whose delivery-tube passes through a water-trap. (The delivery-tube should end well toward the bottom of the calorimeter.)

Directions.—(1) Fill the steam-generator two-thirds full of water and bring the water to the boiling-point. Dry the calorimeter. Determine its weight, W_c, including that of the lid and the thermometer. Fill the calorimeter to a depth of 10 cm. with cold water. Determine the weight, W_{cw}, of the cold water. Take its temperature, T_{cw}, and immediately introduce the delivery-tube into the calorimeter.

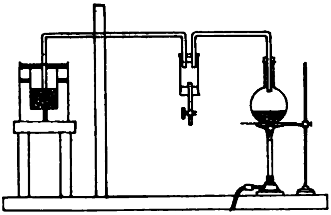

Fig. 51.

(2) As soon as the steam has raised the temperature of the cold water not more than 20° C. above that of the room, remove the delivery-tube from the calorimeter. Before the delivery-tube is removed the water should be stirred with the thermometer, and at the instant the tube is removed the temperature of the mixture, T_m, should be taken.

(3) Weigh the calorimeter with its contents. Take the temperature of the steam, T_s, for the day.

Record data in a neat form.

Calculations.—The theory of this determination is similar to that employed in all the work in calorimetry thus far. That is,

$$\left.\begin{array}{l}\text{Calories lost by steam +}\\ \quad\text{calories lost by steam-}\\ \quad\text{water}\end{array}\right\} = \left\{\begin{array}{l}\text{Calories gained by cold}\\ \quad\text{water + calories gained}\\ \quad\text{by calorimeter.}\end{array}\right.$$

Compute the weight of the steam which was condensed in the calorimeter.

Rewrite the above equation, using letter values.

Substitute numerical values in the rewritten equation, and solve for the heat of vaporization of water. Compare your result with the accepted value.

Queries.—When we say that the heat of vaporization of water is 536 calories, what do we mean? Do different liquids have different heats of vaporization? Do you know of any good reasons why they should have?

CHAPTER VI

MAGNETISM AND ELECTRICITY

EXERCISE 48

Permanent Magnets and Electromagnets *

Apparatus.—Three halves of fine knitting-needles. A bar magnet. A piece of very soft iron. A steel rod. Fine thread. Some magnet wire, No. 24, B. & S. gauge. A battery of several bichromate cells. Some well-annealed wire. Pieces of zinc, copper, glass, wood, and vulcanite.

Preliminary Statement.—On account of the variety of ideas to be presented in this exercise, it may be well to treat them under the following heads:

(A) Magnetic and non-magnetic substances.
(B) Magnetization by the method of contact.
(C) Magnetization by an electromagnet.
(D) Magnetization by induction.
(E) Law of magnets.
(F) Effect of breaking a magnet.

Directions.

Part A.—(1) Determine the character of each substance furnished you, as to whether it is magnetic or non-magnetic.

Part B.—(1) Magnetize a needle by drawing one end of a bar magnet over the needle *twenty* times in one and the same direction. Determine the polarity of both ends of the needle by suspending it. (In this country the *north-seeking* pole is called the north pole.) Mark the end of the needle at which the magnet leaves it. Note the polarity of the end of the bar magnet which was in contact with the needle. *Be able to identify this needle later on.* Call it No. 1. Mark the north pole of this needle by a scratch made with a file.

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* The student will find much of interest relating to magnets in Stewart and Gee, vol. ii. pp. 20–23, and in Hopkins's Experimental Science.

(2) Draw the *north* pole of a bar magnet over another needle (call it No. 2), from the middle to one end, ten times. Draw the *south* pole of the same bar magnet over the needle, from the middle to the other end, ten times. Mark the north pole of this needle.

(3) Determine which needle, if either, is the more strongly magnetized by causing each, in turn, to lift as many fine brads, end to end, as it will.

PART C.—(1) Introduce a steel rod into a helix of suitable proportions. The helix may be laid on its side on the table and then the rod will rest inside the helix without other support. Before currents are passed through the helix, the rod should be at the middle of the helix.

Connect the helix to a battery of several bichromate cells and close the circuit.

(2) Now move the helix to one end of the bar and then to the other end, and finally back to the center. Open the circuit and remove the bar.

(3) Test the magnetic strength of the bar as directed in (3) of Part B.

(4) Repeat (1), (2), and (3) several times with the object of determining whether the first treatment magnetized the bar to the point of saturation.

(5) Repeat (1), (2), (3), and (4), using a rod of very soft iron.

PART D.—(1) Suspend, by means of a strip of paper, in a north and south direction, a needle which has *not* been magnetized. Suspend needle No. 1 by a thread from a support of some sort. When needle No. 1 comes to rest, one pole should be about 2 cm. from one end of the unmagnetized needle.* Now, present a bar magnet to the end of the unmagnetized needle opposite to the end which is presented to needle No. 1. (Do not allow the bar magnet to move or touch the unmagnetized needle.) Observe what happens.

(2) Interpose screens of glass, wood, zinc, copper, vulcanite, and sheet iron between the bar magnet and the unmagnetized needle. Note what happens in each case.

PART E.—(1) Present the south-seeking pole of needle No. 2 to the north-seeking pole of needle No. 1 while it is still suspended. Present the north-seeking pole of needle No. 2 to the north-seeking pole of needle No. 1. Note what happens.

PART F.—(1) Break needle No. 1 into two equal parts. Suspend each half. Determine and mark polarity of both ends of each half. Break each half into two equal parts. Suspend each piece. Determine and mark the polarity of both ends of each. Arrange the pieces in their original order and observe the arrangement of the poles.

CONCLUSIONS.—The student should deduce one or more general principles from each principal division of the exercise and give a summary of these principles in his report.

EXERCISE 49

Lines of Magnetic Force

APPARATUS.—Two bar magnets. A horseshoe magnet. A quantity of fine, soft iron-filings. A piece of window-glass 20 × 25 cm.

PRELIMINARY DISCUSSION.—A line of magnetic force can be defined only in a provisional way. In Exercise 48 the student found that exceedingly small bodies may have a distinct polarity, and that these magnetized bodies tend to line up in a certain manner. Furthermore, when iron-filings are seen to line up in a systematic manner around the poles of a magnet, the conclusion is inevitable that the lines described by the rows of filings delineate a system of force lines in which the magnet acts on the filings. If magnetism may be defined to be a condition in and around the iron due to ether whirls within and around the molecules, then a line of magnetic force may be defined to be the axis of an ether whirl.

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* The two needles should lie as nearly as possible in the same straight line.

This exercise is intended to direct the student's attention to the following points:

(1) The system of distribution of force lines around magnets of different form.

(2) Force lines tend to form closed loops.

(3) Force lines from the same pole, or from like poles, repel each other.

(4) Force lines pass from the north pole of one magnet to the south pole of another magnet, or from the north pole of a magnet to the south pole of the same magnet.

DIRECTIONS.—(1) Place the piece of window-glass on the bar magnet. Sprinkle iron-filings over the glass. Tap the glass with a pencil to facilitate the formation of the filings into lines. Study the system of lines thus produced, with the above-mentioned points in mind.

(2) Repeat (1), using the bar magnets, first with like poles facing each other at a short distance apart, and then with unlike poles facing each other at the same distance apart.

(3) Repeat (1), using a horseshoe magnet.

CONCLUSIONS.—Incorporate in your report a comprehensive summary of your observations touching each point alluded to above. Represent each different system of force-lines by a diagram. Draw this diagram in your note-book, or make a blue-print of the filings and paste the blue-print in your note-book. If you do not know how to make these prints, ask your instructor for directions.

EXERCISE 50

The Simple Voltaic Cell, and Fundamental Ideas concerning the Magnetic Needle used as a Galvanoscope

APPARATUS.—A small compass of good quality. A small beaker, or a tumbler, partly filled with dilute sulphuric acid. Strips of amalgamated and unamalgamated zinc. Strips of copper. A circuit key.

DIRECTIONS.—(1) Place an *unamalgamated* zinc strip in the beaker. Observe what occurs. Place an *amalgamated* zinc strip in the beaker. Again observe what occurs. Also place the copper strip in the beaker. Compare the action of the acid on the different strips.

(2) Now place the copper and the unamalgamated strip of zinc in the beaker, and then the copper and the amalgamated zinc strips. In using each pair make observations in each of the following cases:

(*a*) When they are not connected.
(*b*) When they touch within the liquid.
(*c*) When the outer ends are connected by a wire.

(3) Using copper and amalgamated zinc, perform the following operations:

Connect the copper and the amalgamated zinc to a coil of magnet wire, consisting of ten turns, wound around a compass.* Place the compass so that the turns shall have a north and south direction. Close the circuit and observe what happens. (What is the technical name of the phenomenon observed?) Find the direction of the current in the external circuit by Ampère's Rule.† Notice whether your conclusion accords with the assumption that the current leaves a voltaic cell at the copper terminal. Exchange the wire connections at the terminals of the cell, and again find the direction of the current by Ampère's Rule.

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* Introduce the circuit-key in one line.

† See text-book.

Wind a coil of thirty turns around the compass. Connect the terminals of the coil to a voltaic cell. Compare the deflection with that produced when ten turns were used.

CONCLUSIONS.—Incorporate in your report a comprehensive summary of your conclusions touching the principal points of the exercise.

EXERCISE 51

Law of the Tangent Galvanometer

APPARATUS.—A tangent galvanometer. A commutator.* A resistance box. Several Daniell cells whose resistance is known.

PRELIMINARY DISCUSSION.—In Exercise 50 attention was directed to the fact that the angle of deflection of a magnetic needle depends upon the number of turns composing the coil which acts upon the needle.

We are now prepared to examine another relation—that which the angle of deflection, or some function of it, sustains to the current strength that produces the deflection.

At first thought, the student may be inclined to say that the current strength in a

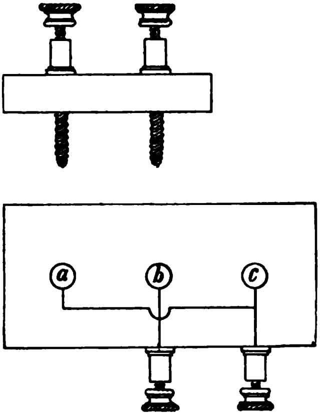

FIG. 52.

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* A very simple commutator is shown in Fig. 52. The holes *a*, *b*, and *c* should be nearly filled with mercury.

given case is proportional to the angle of deflection which it produces. But this is *not* true. Briefly stated, the tangent galvanometer is an instrument so constructed that a given current strength is proportional to the ***tangent*** of the angle of deflection.

In using this instrument, the following points should be observed:

(1) The instrument should be level, and the plane of the coil should coincide with the magnetic meridian.*

(2) When the pointer is at rest its ends should be at, or very near, zero.

(3) No iron should be near the galvanometer.†

(4) Avoid parallax in taking readings of the needle.

(5) The wires which connect the galvanometer with other instruments should be twisted together for some distance from the former.

(6) On account of the fact that the needle may not be straight, or the needle may not come to rest at zero, or the plane of the coil may not coincide with the magnetic meridian, it is necessary in every case to take four readings of the needle: one for each end of the needle with the current flowing in one direction through the coil, and one for each end of the needle with the direction of the current reversed. The mean of these readings will give a value more nearly correct than any one of the four.

DIRECTIONS.—(1) See that the galvanometer is in adjustment, as just described. Connect the instruments ***in series***, as shown in Fig. 53.

(2) Adjust the resistance box until a resistance is obtained which will produce a deflection of about 60°. Take

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* What is the angle made by the intersection of the magnetic and geographic meridians at your laboratory? What is the name of this angle?

† Iron on the person of the experimenter often disturbs the needle. Special hints on this point should be given by the instructor.

the four readings of the galvanometer needle as directed in the preliminary discussion.*

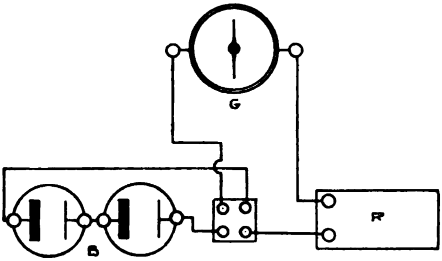

FIG. 53.

(3) Adjust the resistance box until the *total* resistance of the circuit (this includes the resistance offered by the box, the galvanometer, the connecting wires, and the battery itself) is just double what it was before. Record the total resistance used in (2) and in (3). Take the readings of the galvanometer, as before.†

(4) Repeat (2) and (3) several times.

(5) Record the E. M. F. of the battery.

CALCULATIONS AND RESULTS.—Compute the mean angle of deflection for each case. Get the tangents of these angles from a table of tangents to be found in the Appendix.

By Ohm's Law we know that, if the resistance is *doubled* while the E. M. F. remains constant, the current strength will be *halved.* Since the resistance used in (3) was just double what it was in (2), the current strength in the latter case must have been just half what it was in the former case. Express this reasoning in the form of a proportion. Substitute the ratio, 2 : 1, and the corresponding tangents, in the proportion you have framed, and verify.

Knowing the E. M. F. of the battery and the resistance

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*If the galvanometer has more than one coil, find by trial which one, with a *total* external resistance comparable to the resistance of the battery, will produce the required deflection.

†Such coils should be used in this operation as to produce, in conjunction with the external resistance employed, a deflection which is not below 40°. See Stewart and Gee, vol. ii, p. 230.

in each case, we may compute the actual currents and use these computed values instead of the *ratio* of the currents. Compute the current for each case by Ohm's Law. The tangents of the angles of deflection produced by these currents have already been taken from a table of tangents. State the law of the tangent galvanometer in the form of a proportion. Substitute numerical values in this proportion, and verify.

EXERCISE 52

Reduction Factor of a Tangent Galvanometer by Ohm's Law

APPARATUS.—A tangent galvanometer. A commutator. A resistance box. Several Daniell cells freshly made up by formula to have a certain E. M. F.

PRELIMINARY DISCUSSION.—As has been stated, the tangent galvanometer is an instrument so constructed that the current strength is proportional to the tangent of the angle of deflection. That is,

$$C : C_1 :: \tan A : \tan B.$$

In order to use this proportion to compute an unknown current strength, three of the quantities must be known. But, as will soon appear, two of these quantities may be determined once for all, provided neither the instrument nor its adjustment is changed in any way. In making this determination it will be convenient to take a current strength that will produce a deflection of 45°, for the reason that the tangent of 45° is 1.

Let the reduction factor, k, be the number of amperes required to produce a deflection of 45°. Substituting k for C_1, and 1 for $\tan B$ in the above proportion, we have,

$$C : k :: \tan A : 1.$$

$$\text{Or, } C = k \tan A.$$

Whence, if the tangent of the angle of deflection be multiplied by the reduction factor of the instrument used, the product will be the current strength. So, if the reduction factor of a galvanometer is known, it is a fairly convenient instrument for measuring the strength of electric currents.

DIRECTIONS.—(1) See that the galvanometer is in adjustment as described in the preceding exercise. Connect the galvanometer, resistance box, commutator, and battery in series.

(2) Using the galvanometer coil of lowest resistance, adjust the resistance box until the deflection of the galvanometer needle is between 40° and 50°—the nearer it is to 45° the better. Take four readings of the needle as directed in Exercise 51. Record the E. M. F. and resistance of the battery and the external resistance, that of the galvanometer and the resistance box combined.

(3) Proceed as directed in (2), using each coil of the galvanometer in turn.

Record data in the following form:

Galvanometer No. ......

COIL No.	**Resistance of coil.**	**Resistance of cells used.**	**E. M. F. of cells used.**	**Resistance of box.**	**Needle readings.**
1	....	....	....	....	(1)....... (2)....... (3)....... (4)....... *Mean*....
2	....	....	....	....	(1)....... (2)....... (3)....... (4)....... *Mean*
Etc.	Etc.	Etc.	Etc.	Etc.	Etc.

CALCULATIONS AND RESULTS.—Knowing the *total resistance* and the E. M. F. for the case of each coil, compute the current strength by Ohm's Law. This will give you *C* of the equation, $C = k \tan A$. Then compute k for *each coil.* (It can be hardly insisted on too much that each coil has its own reduction factor, and that in the future use of the instrument to measure currents the student must be sure to note which coil is used.)

EXERCISE 53

Reduction Factor of a Tangent Galvanometer by Means of a Copper Voltameter

APPARATUS.—The same as that for Exercise 52, with the addition of a sensitive balance, a copper voltameter, and a solution of copper sulphate of density 1.17.

PRELIMINARY STATEMENT.—When the passage of an electric current of unvarying strength through a standard solution of copper sulphate causes a deposit of 0.000328 gram of copper to appear on the cathode of an electrolytic cell per second, the current strength is said to be 1 ampere. Let it be required to determine the reduction factor of a tangent galvanometer from the weight of copper deposited during a given time.

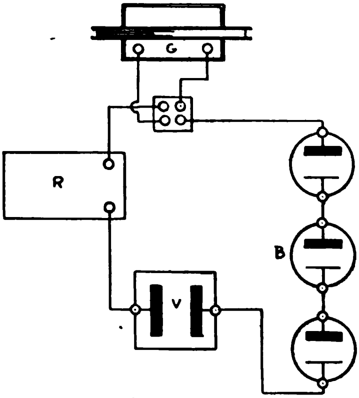

FIG. 54.

DIRECTIONS.—(1) See that the galvanometer is in proper adjustment. Connect the apparatus as directed in the preceding exercise. Before beginning the exercise allow the current to flow through a moderate resistance to get the battery into good working order. (*Do not introduce the voltameter into the circuit yet.*)

(2) Clean the anode and cathode. To do this, first sandpaper them, then wash them in the different cleansing solutions used in copperplating.* Dry the plates in a current of warm air. At no time should the fingers touch the cathode. Now weigh the cathode on a sensitive balance, recording this weight. Place the plates in the voltameter cell, and introduce the voltameter into the circuit.† Close the circuit, and quickly adjust the resistance box, so that the deflection of the galvanometer needle shall be in the neighborhood of 45°. Note the exact time, *to the second*, at which the switch or key is closed.

(3) At intervals of five minutes take four readings of the needle. From these readings compute the mean deflection. Make this computation and record it before the next readings are taken. If, when the next mean deflection has been determined, a change in the current strength is indicated by an increased or a decreased deflection of the needle, adjust the resistance box so as to bring the needle to the position first determined. The deflection of the needle should be as nearly constant throughout the exercise as possible. Take readings every five minutes until at least *twenty* mean deflections have been determined, and make the usual records in proper form.

(4) At the end of the time specified open the circuit and note the exact time at which this is done.

(5) Remove the plates. Wash the *cathode*, first in ordinary clear water, then in distilled water, and finally in alcohol. Dry the cathode in a current of warm air. Weigh the cathode with great care on a sensitive balance. This weight, minus the original weight of the plate, is the number of grams of copper deposited.

Record all data in a tabular form.

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* See Stewart and Gee, vol. ii, p. 70.

† In making these connections be sure that the current will enter the voltameter *at the anode*.

Calculations and Results.—From the twenty mean deflections computed during the progress of the experiment compute the *average* of all the deflections. Call this average the deflection for the entire time. Get the tangent of this average angle from a table of tangents.

From the weight of copper deposited during the experiment, the electrochemical equivalent of copper given in the preliminary statement, and the total time the current was flowing, in seconds, compute the current, C.

Finally, compute the reduction factor, k.

Make all these computations for each coil, or each combination of coils, of the galvanometer used.

EXERCISE 54

Fall of Potential*

Apparatus.—A Daniell battery of three or four cells. A key. Three coils having the respective resistances called for below. A voltmeter or a high-resistance galvanometer.

Preliminary Discussion.—The student should guard against a loose use of the terms electromotive force and difference of potential. The former is the electrical pressure developed *in* the generator. In virtue of this pressure, a current is set up in the entire circuit of which the generator is itself a part. Now, the electromotive force of the generator produces an electrical pressure at *all* points in the circuit. It is self-evident that the pressure at one of these points must be different from that at another, else no current can flow between the points in question. The difference of electrical pressure between two points is known as difference of potential.

Directions.—(1) Connect the apparatus as shown in the figure. Introduce a resistance of 20 ohms between a and b, 40 ohms between b and c, and 60 ohms between c

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* Stewart and Gee, vol. ii, p. 102.

and *d*.* Close the key. Do not open the main circuit or change its resistance throughout the performance of the exercise. Connect the terminals of the voltmeter, or those of the galvanometer, to the main circuit at *a* and *b*. Take the reading of the voltmeter, or of the galvanometer. If the latter instrument is used, see Exercise 51 for directions for reading a tangent galvanometer. Estimate the fractional division in tenths.

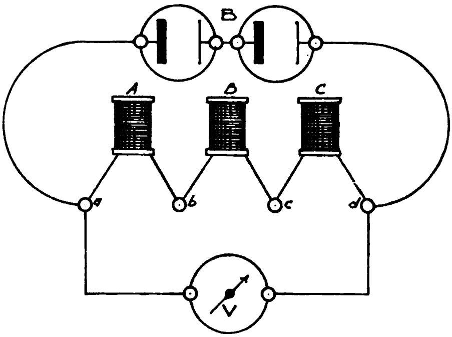

FIG. 55.

(2) *Without changing the resistance of the main circuit,* connect the voltmeter, or the galvanometer, at the points *b* and *c*. Take readings as directed in (1).

(3) Connect the voltmeter, or the galvanometer, at the points *c* and *d*. Take readings as before.

(4) Connect the voltmeter, or the galvanometer, at the points *a* and *d*.

Record data in the following form:

(1) PART OF CIRCUIT.	(2) Fall of potential.	(3) Resistance of part of circuit.	(4) $\frac{\text{Fall of potential}}{\text{resistance}}$.
$a-b$			
$b-c$			
$c-d$			
$a-d$			

---

* The resistances given in (1) are merely suggestive; they should be suited to instrumental conditions, particularly to the sensibility of the voltmeter or galvanometer used.

Calculations and Results.—What relation does the fall of potential—or difference of potential, for these are synonymous expressions—between two given points bear to the resistance between the same points? Frame a law which shall express the meaning of the results given in column (4) of the table. Express this law in the form of a proportion. Substitute in the proportion you have framed, and verify.

Using resistances as abscissas and potential differences as ordinates, plot a curve to represent the relation between them. Observe that this curve shows graphically the fall of potential from point to point.

Queries.—What does the voltmeter measure? Its coils should have a very high resistance: why? Suppose the voltmeter terminals to be connected to two points, X and Y, between which the resistance of the conductor is zero. (*a*) What is the difference of potential between these points? (*b*) Explain your answer to (*a*) in terms of the proportion you have framed as directed above. The resistance of the feeder wires used on electric railways is very small: what can you say of the difference of potential between the terminals of such a line? Why is a low resistance of the feeder wires a desirable thing?

EXERCISE 55

Verification of Ohm's Law

Apparatus.—A Daniell battery of several cells. A key. A resistance box. A voltmeter and an ammeter; or, high and low resistance galvanometers. The constants of the latter instruments should be known.

Directions.—(1) The instruments are to be connected essentially as shown in Fig. 55. Put the resistance box in the middle branch. Connect the ammeter, or the low-resistance galvanometer, and the key in series with the battery. Connect the voltmeter, or the high-resistance galvanometer, in the lower branch.

(2) Remove a 10-ohm plug from the resistance box. Close the key and take simultaneous readings of the voltmeter and ammeter, or of the galvanometers if the latter instruments are used. (One student should read the voltmeter while another reads the ammeter. This should be done quickly upon the giving of a prearranged signal; for example, when one student calls, "Now!")

If, when no current is flowing through the instruments, either needle does not stand at zero, read its position very carefully and correct all readings by adding or subtracting the amount by which the needle fails to come to rest at zero. If galvanometers are used, take readings as directed in Exercise 51.

(3) Replace the 10-ohm plug and remove a 5-ohm plug. Take readings as directed in (2).

(4) Replace the 5-ohm plug and remove a 1-ohm plug. Take readings as before.

Calculations and Results.

(I) *For the voltmeter and ammeter method.* The resistance of the ammeter and the connecting wires is so small as to be negligible. The resistance of the circuit consisting of the upper and middle branches (see Fig. 55) is therefore the resistance introduced by the box and that of the battery.

The number of amperes of current, C, flowing through the circuit just described is measured by the ammeter.

The difference of potential, E, between the points a and d is measured by the voltmeter.

For the partial circuit in question, we now have all the quantities involved. By Ohm's Law, these quantities should be related in the following manner:

$$C = \frac{E}{R}.$$

(II) *For the galvanometer method.* Consider the middle branch. The resistance of this branch is given by the resistance box.

The current, C, flowing through this branch can be determined from the deflection of the low-resistance galvanometer, if the reduction factor of the latter is known. (See Exercise 52.) As is shown in the exercise just referred to, if the *tangent* of the observed angle of deflection is multiplied by the reduction factor, k, of the galvanometer used, the product will be the number of amperes of current flowing through the branch in question, and hence through the middle branch.

The difference of potential, E, between the extremities of the middle branch can be determined from the deflection of the high-resistance galvanometer by means of the following equation:

$$E = k\ (\tan A)\ R,$$

where k is the reduction factor of the galvanometer used, $\tan A$ is the tangent of the observed angle of deflection, and R is the resistance of the middle branch.

Compute C and E as just directed. Substitute numerical values of C, E, and R in Ohm's Law, and verify.

EXERCISE 56

Measurement of Resistance by the Method of Substitution

APPARATUS.—A galvanoscope. A commutator. A coil of wire whose resistance is to be measured. A resistance box. A battery of Daniell cells. A key or switch.

DIRECTIONS.—(1) Connect the apparatus in series. Adjust the position of the galvanoscope until the north pole of the needle comes to rest at the zero point of the scale. Do not disturb the instrument throughout the exercise.

(2) Close the switch. When the needle comes to rest, read the position of both ends of the needle. Reverse the

switch and read as before. Take the mean of the four readings.

(3) Open the switch. Remove the coil from the circuit, and in its place connect a resistance box, *R*. See that the plugs are firmly in place in the box. Now close the switch. Remove the 20-ohm plug as a trial. If this causes a greater deflection than that obtained when the coil was in the circuit, replace the plug, and remove one of higher resistance. If, on the contrary, the 20-ohm resistance causes a smaller deflection, replace it and introduce less resistance into the circuit. Use plugs marked tenths for the fine adjustment. When a mean deflection is obtained which is equal to the one observed when the coil was in the circuit, add up the numbers opposite the plugs which have been removed from the box. The resistance indicated by the box is concluded to be that of the coil.

CONCLUSIONS.—Explain the principle of this method. Does the accuracy of this method depend upon the battery? Explain why, or why not, as the case may be.

EXERCISE 57

Measurement of Resistance with the Slide-wire Bridge: Laws of Length and Cross-section

APPARATUS.—A slide-wire bridge. A commutator. A key or switch. A resistance box. Three coils of wire, *a*, *b*, and *c*. Coils *a* and *b* have different lengths but the same diameter, while coils *b* and *c* have the same length but different diameters. (These lengths and diameters should be given the student to save time, as little is gained by having him measure them for himself.)

LAWS TO BE VERIFIED.—(1) For the case of homogeneous conductors of uniform cross-section, the resistance of a conductor is directly proportional to its length.

(2) For the case of homogeneous cylindrical conductors

having the same length but different cross-sections, the resistance of a conductor is inversely proportional to the square of its diameter.

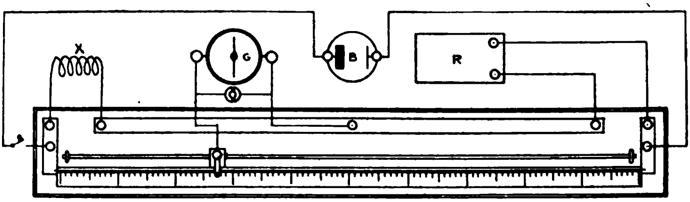

Fig. 56.

THEORY OF THE SLIDE-WIRE BRIDGE.—Let *AFB* and *ADB* be branches constituting the divided portion of a given closed circuit. Let *A* be at a higher potential than *B*. Since *F* lies between *A* and *B*, its potential must be lower than that of *A* and higher than that of *B*. Now every possible potential between those of *A* and *B* may be found on the branch *ADB*. Therefore, some point on it, as *D*, may be found, whose potential is the same as that of the point *F*, because every possible potential between those of *A* and *B* are also found on the branch *ADB*. Hence, if the points *F* and *D* be electrically connected, no current will flow from *F* to *D* or from *D* to *F*.

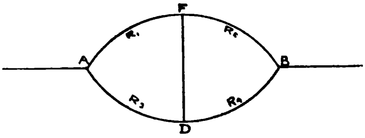

Fig. 57.

PROOF OF THE LAW OF THE BRIDGE.—Let R_1, R_2, R_3, and R_4 be the resistances, respectively, of the parts of the circuit, *AF*, *BF*, *AD*, and *BD*. Also, let C be the current flowing in the branch *AFB*, and C_1 that in the branch *ADB*.

By Ohm's Law the fall of potential from *A* to *F* is equal to R_1C. For the same reason, the fall of potential from A to D is equal to R_3C_1. Again, the fall from F to B is equal to R_2C, and that from D to B is equal to R_4C_1.

Whence, $R_1C = R_3C_1$. (1)

And, $R_4C_1 = R_2C$. (2)

Multiplying (1) by (2), we have

$$R_1R_4CC_1 = R_2R_3CC_1.$$

Whence, $R_1R_4 = R_2R_3$. (3)

In using the bridge, the following general directions should be observed:

(1) See that all connections are properly and securely made, using as heavy conductors as the case will permit.

(2) Introduce into the R_1 arm of the bridge the resistance to be measured and into the R_2 arm a resistance comparable to that of the former.

(3) Close the key temporarily and rapidly change the resistance of the R_2 arm—that of the resistance box—until the galvanoscope needle falls back approximately to zero.

(4) After having roughly balanced the bridge, secure a more perfect balance by moving the slider of the bridge along to such a position that, when the key is closed, *no* deflection of the needle can be observed. What condition has now been attained?

(5) When the bridge is perfectly balanced, the products of the *opposite* arms are equal, as indicated by equation (3).

DIRECTIONS.—(1) Make connections as indicated by the figure, introducing coil *a* into the R_1 arm of the bridge. Balance the bridge. Record the resistance introduced into the R_2 arm of the bridge—that of the box. Measure and record the lengths of the arms AD and BD. These lengths are the distances, respectively, from the bridge key to the left-hand or A end of the bridge, and from the key to the right-hand or B end of the bridge.*

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* Compare Figs. 56 and 57.

(2) Make the necessary changes and connections, using coil *b*. Proceed as directed in (1).

(3) Make the necessary changes and connections, using coil *c*. Proceed as directed in (1).

CALCULATIONS AND RESULTS.—In order to use the slide-wire bridge in proving Law I, we must assume the law as proved in so far as it relates to the bridge-wire—that is, in order to use the bridge at all, we must assume that the resistance of any portion of the wire is proportional to the length of the portion in question. The only justification for assuming the very proposition which is to be proved is the fact that Law I is almost self-evident. The really important points in the exercise are the verification of Law II, and practice in the use of the bridge.

The *lengths* of the arms AD and BD may therefore be substituted for R_3 and R_4, respectively. Substitute these lengths, and the resistance introduced by the box R_2 in equation (3), and solve for R_1. In this manner compute the resistance of coils *a*, *b*, and *c*.

State Law I in the form of a proportion. Substitute numerical values in the proportion you have framed, and verify.

State Law II in the form of a proportion. Substitute in this proportion, and verify.

EXERCISE 58

Joint Resistance of Branched Circuits

APPARATUS. — Three coils of different resistances, mounted and connected as shown in Fig. 58. A slide-wire bridge with the necessary accessories.

PRELIMINARY DISCUSSION.—Bear in mind, first of all, that the joint resistance of a circuit consisting of *any number* of branches is less than the resistance of the branch having the least resistance. The joint cross-section of such a circuit is greater than that of any one of the branches. Hence, the joint resistance of a branched circuit is less than the resistance of any branch.

The conductivity of a circuit whose resistance is 1 ohm is taken as 1. Hence, the conductivity of any circuit is the *reciprocal* of the resistance of the circuit, and *vice versa.*

Again, the joint conductivity of a circuit consisting of n branches is equal to the sum of the conductivities of the several branches. *The resistance of such a circuit is the reciprocal of its joint conductivity.*

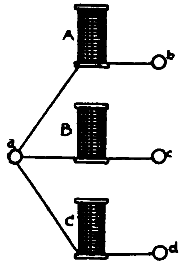

FIG. 58.

PROPOSITIONS.—Let it be required to measure the joint resistance of the following branched circuits:

(I) That of a circuit consisting of two branches. Let this be the joint resistance of the branches, aAb and aBc, as shown in the figure.

(II) That of a circuit consisting of three branches. Let this be the joint resistance of the branches, aAb, aBc, and aCd, as shown in the figure.

DIRECTIONS.—(1) Connect the terminals of coil A to the X or R_1 arm of the bridge. Measure the resistance of coil A with great care.

(2) Disconnect coil A. Connect coil B to the R_1 arm of the bridge. Measure the resistance of coil B.

(3) Disconnect coil B. Connect coil C to the R_1 arm of the bridge. Measure the resistance of coil C.

(4) Disconnect coil C. Connect, *in multiple*, the terminals of coils A and B to the R_1 arm of the bridge. Measure the joint resistance of coils A and B.

(5) Connect, *in multiple*, the terminals of coils A, B, and C to the R_1 arm of the bridge. Measure the joint resistance of coils A, B, and C.

Record data in tabular form.

CALCULATIONS AND RESULTS.—Compute the resistance of each coil from data obtained in (1), (2), and (3).

Compute the joint resistance of coils *A* and *B*; also of coils *A*, *B*, and *C*. Use data obtained in (4) and (5).

Write an equation for computing the joint resistance of a circuit consisting of *two* branches.

Write an equation for computing the joint resistance of a circuit consisting of *three* branches.

Substitute in these equations and verify.

EXERCISE 59

Specific Resistance

APPARATUS.—A slide-wire bridge with the necessary accessories. Coils of copper and German-silver wire. The length and diameter of the wire composing these coils should be known in advance.

PRELIMINARY DISCUSSION.—The resistance of a conductor varies with the material of which it is composed. The specific resistance of a given substance is usually defined to be the resistance of a block of the substance having a length of 1 cm. and a cross-section of 1 square cm. For present purposes the following definition is sufficient:

The specific resistance of a substance is the resistance of a conductor of that material having a length of 1 meter and a constant diameter of 1 mm.

DIRECTIONS.—(1) Measure and record the resistance of the copper wire. Record its length and diameter.

(2) Measure and record the resistance of the German-silver wire. Record the length and diameter of the German-silver wire.

(3) Repeat (1) and (2) several times.

CALCULATIONS AND RESULTS.—The resistance of a copper wire having a given length and diameter has been determined in the experiment. From these three quantities the resistance of a copper wire whose length is 1 meter and whose diameter is 1 mm. can be computed. The resistance last mentioned will be the specific resistance of the copper furnished you, according to the definition of specific resistance given in the latter part of the preliminary discussion. The foregoing remarks apply with equal force to the German-silver wire.

EXERCISE 60

Temperature Coefficient of Resistance of Copper

APPARATUS.—A slide-wire bridge with its accessories. A coil of fine copper wire. A vessel in which to suspend the coil. Ice-chips. A thermometer.

PRELIMINARY DISCUSSION.—The resistance of conductors varies with the temperature. For most substances the rate of this change of resistance is fairly uniform through moderate ranges of temperature. The rate, per degree centigrade, at which resistance increases or decreases is called the *temperature coefficient of resistance.* It is always a fraction. It is the fraction of the resistance of a given conductor at 0° C., by which its resistance is increased or decreased when the temperature is changed 1° C.

For example, the temperature coefficient of pure copper is 0.00388. This means that a pure copper conductor whose resistance at 0° C. is 50 ohms will have its resistance increased 0.00388 of 50 ohms if its temperature be raised 1° C. If its temperature be raised 35° C., then the resistance of the conductor will be,

$$50 + (50 \times 0.00388 \times 35) = 56.79 \text{ ohms.}$$

DIRECTIONS.—(1) First place the copper coil in a suitable vessel, packing it around with ice-chips, and finally filling the vessel with water. Connect the terminals of the coil with the X-arm of the bridge. Stir the ice-chips and water for two minutes. Then close the circuit and measure the resistance of the coil.

(2) Remove it to a vessel containing boiling water, and after two minutes proceed to measure the resistance of the coil as in (1).

(3) Compute the temperature of the boiling water from the barometric pressure.

(4) Measure the resistance of the coil at several points between 0° C. and 100° C. Take the temperature at the instant the bridge is balanced. Record each resistance and the corresponding temperature.

Calculations and Results.—From the definition of the temperature coefficient of resistance and the example given above, form an equation for computing the coefficient. Substitute results obtained in (1), (2), and (3) in the equation, and solve for the coefficient. Using temperatures as abscissas and resistances as ordinates, plot a curve showing the relation of resistance to temperature for the case of copper. Determine approximately the point at which the curve would intersect the temperature axis if the latter were extended sufficiently to the left. To what temperature does the point just mentioned correspond? What very interesting inference may be drawn from this fact?

EXERCISE 61

Resistance of a Voltaic Cell

Apparatus.—One or more cells whose resistance is to be measured.* A resistance box. A voltmeter. A tangent galvanometer whose resistance and reduction factor are known. A commutator.

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* The number of cells to be used will be determined by the sensitiveness of the voltmeter and galvanometer used.

PART A.—**Fall of potential method.**

DIRECTIONS.—(1) Measure, on open circuit, the E. M. F. of the cell or cells to be tested. Call this difference of potential *E*.

(2) Connect the apparatus as indicated by Fig. 59. (The commutator is not shown.) Remove a plug from the resistance box, which will introduce a resistance, *R*, comparable to the resistance of the battery under test. Take the voltmeter reading, *V*.

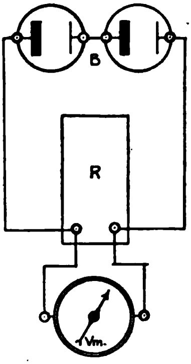

FIG. 59.

(3) Repeat (1) and (2) several times, recording each set of values.

PART B.—**Method of Ohm's Law, using a tangent galvanometer.**

DIRECTIONS.—(1) Connect the apparatus as shown in Fig. 60. Introduce resistance into the circuit until a deflection of between 55° and 60° is produced. Take direct and reversed readings of the galvanometer. Record the box resistance and that of the galvanometer, calling their sum *R*.

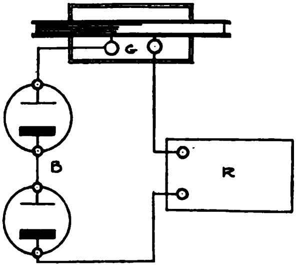

FIG. 60.

(2) Introduce resistance into the circuit until a deflection of between 40° and 45° is produced. Take direct and reversed readings of the galvanometer. Record the box resistance and that of the galvanometer, calling their sum R_1.

(3) Repeat (1) and (2).

CALCULATIONS AND RESULTS.

PART A.—If *E* is the difference of potential between the terminals of the battery on open circuit, *R* is a given external resistance, and r is the resistance of the battery, then, by Ohm's Law,

$$C = \frac{E}{R + r}. \qquad (1)$$

Now if a voltmeter be attached to the terminals of the resistance box and the E. M. F. of the battery and the resistance of the entire circuit does not change, the current C (which is C of (1)) will be given by the equation,

$$C = \frac{V}{R}, \qquad (2)$$

in which V is the difference of potential at the terminals of the box.

Whence, from (1) and (2),

$$r = \frac{R(E - V)}{V}. \qquad (3)$$

Substitute sets of related values in (3) and solve for r.

PART B.—If E is the E. M. F. of the battery as determined in (1) of Part A, R is the total external resistance, and r is the resistance of the battery, then,

$$C = \frac{E}{R + r}. \qquad (1)$$

If R be changed to R_1, and the E. M. F. and the resistance of the battery do not change, then the current, C_1, will be given by the equation,

$$C_1 = \frac{E}{R_1 + r}. \qquad (2)$$

Whence, dividing (1) by (2) and equating for r, we have,

$$r = \frac{C_1 R - C R_1}{C - C_1}. \qquad (3)$$

Compute the mean deflection for each case. Then compute C, the current flowing in the circuit under the conditions as specified in (1). Also compute C_1, the current flowing in the circuit under conditions as specified in (2).

Substitute in equation (3) and solve for r. If more than one cell was used, compute the resistance per cell.

Compare the values of r as obtained by the two methods employed in this exercise.

EXERCISE 62

Resistance of a Voltaic Cell by Mance's Method

APPARATUS.—A slide-wire bridge with its accessories. A Daniell or a gravity cell whose resistance is to be determined.

DIRECTIONS.—(1) The several parts of the apparatus are to be connected essentially as shown in Fig. 56, with certain slight changes. The cell whose resistance is to be measured should be connected to the R_1, or X, arm of the bridge. The line which usually includes the battery should be replaced by a heavy wire, provided with a circuit-key. The galvanometer is connected as usual. *Its key should remain closed.*

(2) When connections have been made as just described, the galvanometer needle will be permanently deflected. If this deflection is very much greater than 45°, it should be reduced by shunting the galvanometer. (A convenient shunt is a resistance box connected to the terminals of the galvanometer.) A controlling magnet may be used to effect the same result. The magnet should be placed as far away from the galvanometer as possible, so as not to reduce the sensibility of the galvanometer too much.

Adjust the position of the contact key on the bridge-wire so that, when the key in the usual battery line is closed, the deflection of the needle will not *materially* change. In making this adjustment, a *marked* change of the position of the needle will indicate that the correct position of the contact key has not been found. No attention should be paid to a *gradual* change of the position of the needle. Owing to the polarization of the cell, a gradual drift of the needle is likely to be observed. There may also be a slight disturbance of the needle due to self-induction in the bridge coils.

(3) When the closing of the key in the battery line does not produce a decided change in the position of the needle, the points F and D (see Exercise 57) are at the same potential, and the bridge is balanced. Take the reading of the bridge arms, AD and BD.

(4) Repeat (2) and (3) several times.

CALCULATIONS AND RESULTS.—Since, in this experiment, the cell occupies the R_1 arm of the bridge, the resistance of the cell must therefore be R_1, as treated in Exercise 57. Hence, by the law of the bridge,

$$R_1 : R_2 :: AD : BD,$$

where R_2 is the box resistance.

Compute the resistance of the cell. Compute the mean of the different determinations.

QUERIES.—Should the resistance of the shunt referred to in (2) be greater or less than the resistance of the galvanometer? Why? How should the controlling magnet be placed to reduce the deflection of the galvanometer? Why will polarization of the cell produce the effect mentioned in (2)? Why will self-induction in the bridge coils cause a deflection of the needle?

EXERCISE 63

Electromotive Force of a Voltaic Cell

APPARATUS.—A voltmeter. A tangent galvanometer whose reduction factor is known. A resistance box. A circuit-key. A commutator. A Daniell or a gravity cell.

Preliminary Statement.—It is the purpose of this exercise to compare the E. M. F. of a voltaic cell on open circuit with the E. M. F. of the same cell determined by means of a tangent galvanometer.

Directions.—(1) Determine the E. M. F. of the cell on open circuit by means of the voltmeter. In case the voltmeter does not read to tenths of volts, it will be better to dispense with it. In such an event use a cell freshly made up by formula to have a certain voltage. Allow the cell to send a current through a moderate resistance for some time before using it. Take the *rated* E. M. F. for the E. M. F. of the cell *on open circuit.*

(2) Connect the cell, resistance box, commutator, key, and galvanometer in series. Close the circuit-key and adjust the resistance box until a deflection of about 40° is produced. Take direct and reversed readings of the galvanometer. Call the resistance of the box R.

(3) Change R to R_1, a resistance that will produce a deflection of about 55°. Take the galvanometer readings as before.

(4) Repeat (1), (2), and (3), adjusting the resistance box so as to have deflections intermediate between those mentioned in (2) and (3).

Record data in the following form:

Determination.	Box resistances.	Galv. readings.	Current strength.	E. M. F. by voltmeter.	E. M. F. by galv.
I	R........	(1)....... (2)....... (3)....... (4)...... *Mean*....	C........	E........	E........
	R_1.......	(1)....... (2)....... (3)....... (4)....... *Mean*....	C_1.......		
II	Etc.	Etc.	Etc.	Etc.	Etc.

Calculations and Results.—Compute the mean deflection of the galvanometer needle for case. Then compute the current strengths, C and C_1, for each determination. From these current strengths, and from the resistances, R and R_1, the E. M. F. of the cell can be computed. In order to understand how this is done, consider the following:

If E is the electromotive force of the cell, R is a given resistance introduced into the circuit, and R_2 is the combined resistance of the cell, galvanometer, and connections —which may remain unknown, as will presently appear— then, by Ohm's Law,

$$C = \frac{E}{R + R_2}. \qquad (1)$$

If R be now changed to R_1 while E and R_2 remain constant,

$$C_1 = \frac{E}{R_1 + R_2}. \qquad (2)$$

From equations (1) and (2) a single equation may be derived from which R_2 is eliminated. The equation referred to may be stated as follows:

$$E = \frac{CC_1(R - R_1)}{C_1 - C}. \qquad (3)$$

Substitute related sets of values in equation (3) and solve for E. Compare this value of E with that obtained by means of the voltmeter.

Include in your report a complete derivation of equation (3) from equations (1) and (2).

EXERCISE 64

Electromotive Force of a Voltaic Cell in Terms of a Daniell Cell by the High-Resistance Method

Apparatus.—A sensitive tangent galvanometer whose reduction factor is known. A resistance coil of at least 5,000 ohms. A commutator. A resistance box. A Daniell battery of several cells freshly made up by formula to have a specified voltage. A Leclanché battery of several cells.

Directions.—(1) Connect the Daniell battery, the galvanometer, and the 5,000-ohm coil in series. Connect the resistance box to the terminals of the galvanometer, to be used as a shunt if necessary. Adjust the apparatus until a deflection of about 45° is obtained. Take direct and reversed readings of the galvanometer. Record the mean of these readings as the angle of deflection.

(2) Connect the Leclanché battery, the galvanometer, and the 5,000-ohm coil in series. Adjust the apparatus until a deflection is obtained which is about the same as that observed in (1). Take direct and reversed readings of the galvanometer, and record their mean as the new angle of deflection.

(3) Repeat (1) and (2) as a check upon your results.

Calculations and Results.—The interesting feature of this exercise is the fact that by using this method the E. M. F. of a voltaic cell can be computed without knowing either the external or the internal resistance, *provided* that we know that the former is *very large* in comparison with the latter. The reasoning is as follows:

Let E be the E. M. F. of each Daniell cell and E_1 be that of each Leclanché cell. Let r be the resistance of each Daniell cell and r_1 be that of each Leclanché cell. Also, let R be the total external resistance, including that of the 5,000-ohm coil, the galvanometer, and the leads—*all of which may remain unknown.*

With the cells of the Daniell battery in circuit, as directed in (1), the current, C, will be given by the equation,

$$C = \frac{nE}{R + nr}, \qquad (1)$$

where n is the number of cells.

With the Leclanché battery in circuit, as directed in (2), the current, C_1, will be given by the equation,

$$C_1 = \frac{nE_1}{R + nr_1}. \tag{2}$$

Whence, dividing (1) by (2), we have

$$C : C_1 :: \frac{nE}{R + nr} : \frac{nE_1}{R + nr_1}. \tag{3}$$

But R is exceedingly large in comparison with either nr or nr_1. Therefore, for all practical purposes, $R + nr = R + nr_1$. This being so, equation (3) may be written as follows:

$$C : C_1 :: nE : nE_1. \tag{4}$$

Compute C and C_1. If the Daniell cells are freshly made up by formula as directed, their E. M. F. will be known. It only remains to substitute in (4) and solve for E_1, the E. M. F. of a Leclanché cell.

EXERCISE 65

Electromotive Force of Generators connected in Different Ways; Effect of Method of Connection upon Current furnished when External Conditions are Constant

APPARATUS.—Six Daniell cells whose resistance is known. A resistance box. A commutator. A voltmeter.

PRELIMINARY STATEMENT.—Before beginning this exercise the student should know the meaning of the terms, simple series, simple multiple, series-multiple, and multiple-series.

DIRECTIONS.—Measure the E. M. F. of each cell on open circuit. Call these E. M. F.'s, E_1, E_2, E_3, E_4, E_5, etc.

(2) Connect two cells in simple series with the voltmeter. Measure the combined E. M. F. of the two cells.

(3) Connect three cells in simple series. Measure their combined E. M. F.

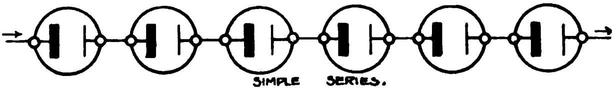

FIG. 61.

(4) Proceed in the same way, adding one cell at a time, measuring the combined E. M. F. in each case.

(5) Connect all the cells in series-multiple. Measure the E. M. F. of the battery.

(6) Repeat (1), (2), (3), and (4), connecting the cells in simple multiple. Measure the E. M. F. of the battery in each case.

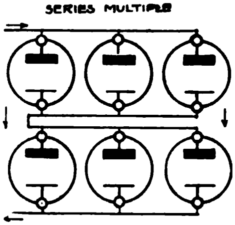

FIG. 62.

(7) Connect all the cells in multiple-series. Measure the E. M. F. of the battery.

(8) Connect six cells in series-multiple—in two rows, with three cells in each row. Compute the resistance of the battery. Introduce in series with the battery a galvanometer and a resistance box. Adjust the box and use such a coil, or coils, of the galvanometer that their combined resistance shall equal the resistance of the battery. Close the circuit and take direct and reversed readings of the galvanometer.

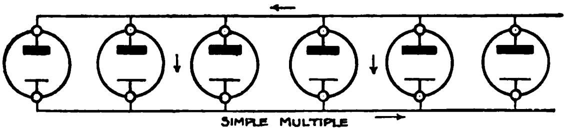

FIG. 63.

(9) Change the connection of the battery cells to multiple-series. *Do not change conditions in the galvanometer or the resistance box.* Take the galvanometer readings as before.

(10) Change the connections of the battery cells, in turn, to simple series and simple multiple. Observe the caution given in (9) with respect to the galvanometer and resistance box. Take the galvanometer readings.

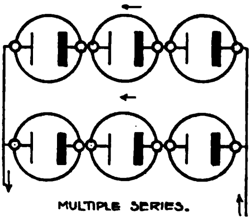

FIG. 64.

Record data in tabular form.

CONCLUSIONS.—From the results of operations (1) to (7), inclusive, deduce a rule for computing the E. M. F. of batteries connected in the different ways specified above.

From the galvanometer deflections observed in (8), compute the mean deflection. Then compute the current strength. Proceed similarly with respect to (9) and (10). Compare the three results.

Observe that in (8), (9), and (10) we are dealing with *fixed* external conditions, with *variable* internal conditions. What general conclusion can you draw from the results of operations (8), (9), and (10)? What practical application is made of the principles involved in the latter part of this exercise?

PROBLEMS.—Suppose three dynamos to be connected in simple series. Suppose the E. M. F. developed at the terminals of the machines are 116 volts, 118 volts, and 114 volts, respectively. What will be the difference of potential between the extremities of this "battery" of dynamos?

Suppose the same machines to be connected in simple multiple. What will be the E. M. F. of the current furnished by this "battery" of dynamos?

EXERCISE 66

(Alternative with Exercise 67)

Current Strength, Wattage, and Horse-power necessary to operate Certain Electrical Instruments

Apparatus.—A key. A tangent galvanometer whose reduction factor is known. A resistance box. An Edison-Lalande battery of several cells, or a bichromate battery freshly made up by formula to give a certain voltage. An electric bell. A telegraph sounder. A telegraph relay. A small electric motor.

Preliminary Discussion.—Even after the student has become fairly well grounded in the elementary principles of electricity, it is quite possible that he may not yet clearly apprehend the relation which exists between the three principal units, the *volt*, the *ohm*, and the *ampere.* To guard against this possibility is the object of this exercise.

Let it be required, therefore, to determine the current strength, the wattage, and the horse-power necessary to operate the above-named instruments.

Résumé of principles involved:

(1) Ohm's Law, $C = \frac{E}{R} = \frac{\text{Volts}}{\text{Ohms}}$.

(2) Watts $= CE$.

(3) Horse-power $= \frac{CE}{746}$.

Directions.—(1) Record the rated E. M. F. of the battery you are to use.

(2) Record the resistance of each instrument which is to be operated. (The student should not be required to measure these resistances unless there is an abundance of time.)

(3) Connect the key, tangent galvanometer, electric bell, commutator, and battery in series. Use just enough cells to operate the bell with the proper degree of loudness.

Close the key and take direct and reversed readings of the galvanometer.* Check this result by repeating this operation several times. Open the key as soon as the readings are taken.

(4) Repeat (3), using the telegraph sounder.

(5) Repeat (3), using the relay. *Use but one cell*, and introduce resistance until the relay will just barely operate. See that the tension spring is very loose, and that the armature works freely.

(6) Repeat (3), using a small electric motor. If sufficient time can be given to the exercise, measure the current consumed (*a*) when the armature of the motor is held stationary while the current is flowing through the instrument; (*b*) when the motor is running under *no load*; (*c*) when the motor is running under a small load; and (*d*) when the motor is pulling a load that reduces its speed in a marked degree.

Record data in the following form:

Instrument Operated.		Rated E. M. F. per Cell........							
		Resistance.	No. cells used.	Rated E. M. F. of battery.	Computed E. M. F. of battery.	Deflection.	Current strength.	Watts.	Horse-power.
Bell.									
Sounder.									
Relay.									
Motor.	Armature held.								
	No load.								
	Small load.								
	Overload.								

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* The vibrator of the bell should be held against the screw, else the self-induction in the coils of the bell will disturb the needle of the galvanometer.

CALCULATIONS AND RESULTS.—Record the E. M. F. of the battery used in each part of the exercise. The E. M. F. of the battery no doubt changed during the progress of the experiment. Therefore, it will be well to *compute* it for each part of the exercise.

By Ohm's Law, $C = \frac{E}{R}$. Whence, $E = CR$.

From the law of the tangent galvanometer, $C = K \tan A$, compute the current strength for each case. Then compute the E. M. F. for each case from the formula, $E = CR$.

Finally, compute the wattage and the horse-power for each part of the exercise.

Do not fail to contrast the horse-power necessary to operate the relay with that necessary to operate the sounder or the electric bell. This comparison should forcibly emphasize the reason for using relays on telegraph lines.

EXERCISE 67

Electrical Horse-power

APPARATUS.—A voltmeter. An ammeter. Incandescent electric lamps of 16 c. p., 50 c. p., and 100 c. p. A small electric motor. An arc lamp.

PRELIMINARY DISCUSSION.—The unit of electrical activity is the watt. The electrical activity—i. e., the wattage—necessary to operate a given instrument or machine is given by the equation,

$$W = CE,$$

where W is the wattage, C is the current, and E is the electromotive force.

The mechanical equivalent of the watt is $\frac{1}{746}$ of a horse-power. Hence the horse-power of a current may be readily computed. In determining the horse-power necessary to operate a given electrical instrument or appliance, under stated conditions, it is only necessary to know the current, C, flowing through the instrument and the difference of potential, E, between its terminals. Let it be required to determine the electrical horse-power necessary to operate the following: (1) A 16 c. p. incandescent lamp. (2) A 50 c. p. incandescent lamp. (3) A 100 c. p. incandescent lamp. (4) An arc lamp. (5) An electric motor running under different loads.

Directions.—(1) Connect the 16 c. p. lamp and the ammeter in series to the mains. Connect the voltmeter to the terminals of the lamp. Now, take a set of simultaneous readings of the voltmeter and the ammeter. A set of at least ten observations of C and E should be taken.

(2) Proceed in the same manner with the other lamps.

(3) Connect the ammeter in series with an electric motor which may be caused to run with various loads from no load to overload. Connect the voltmeter to the terminals of the motor.

(4) Take a set of these readings with no load.

(5) Put on half the rated full load. Take the readings as before.

(6) Put on the full load and take the readings.

(7) Put on additional load until the speed has been considerably reduced. Take the readings.

(8) Turn off the current from the motor. Fasten the armature so that it can not rotate. Now, turn on the current and take the readings. (Do not leave the current on longer than is necessary to take the readings.)

Record data in a table that will present in the most forcible manner the results of this exercise.

Calculations.—Compute the electrical horse-power for each case, using mean values. Incorporate in your report a discussion of any general or special ideas to which this exercise may lead.

CHAPTER VII

SOUND

EXERCISE 68

Vibration Frequency of a Tuning-Fork

APPARATUS.—Long strips of smoked glass to be fastened to a carrier. A tuning-fork mounted in such a way that the stylus attached to one prong of the fork may trace a line on the smoked glass when the carrier is moved along. A short pendulum so mounted that a stylus attached to its lower end may trace lines on the smoked glass perpendicular to its direction of motion.

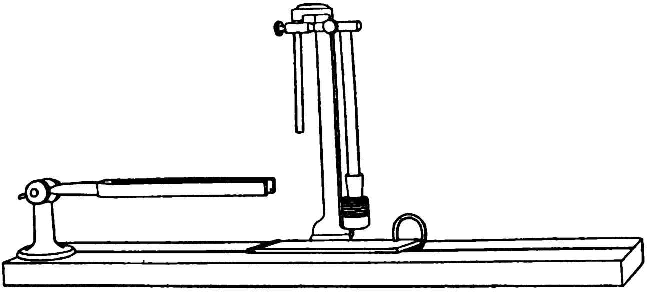

FIG. 65.

PRELIMINARY DISCUSSION.*—By a complete vibration of a body is meant the complete motion which it executes between two successive recurrences of the same position or condition. Such vibrations sometimes are called double vibrations, the number of them per second being known as the vibration frequency, and the time of one of them as the period of vibration.

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* Deschanel, Acoustics, p. 1; Hastings and Beach, p. 571.

DIRECTIONS.*—(1) Set the pendulum swinging across one end of the strip of smoked glass. See that its stylus traces but one line. Strike a prong of the fork with a small mallet, or pass a rosined bow over it. At once draw the slider along, thus causing the stylus to trace a wavy line which, at intervals, will be crossed by lines traced by the pendulum stylus.

(2) Determine the time of one double swing of the pendulum used.

(3) Make several records on different strips of glass.

CALCULATIONS.—Count the number of waves between the most widely separated pendulum marks which enclose an *uneven* number of spaces. The interval of two adjacent spaces evidently represents the time occupied by a complete vibration of the pendulum. Calculate the number of complete vibrations made by the fork in the time of one complete vibration of the pendulum. Knowing the period of the pendulum and the number of vibrations made by the pendulum in this time, the frequency of the fork can be computed.

EXERCISE 69

Velocity of Sound-Waves in Air

APPARATUS.—A tube of glass or other material. A clamp and support to hold the tube at any depth in a vessel of water. Or, better still, if the tube is provided with a stop-cock at its lower end, the height of the column in the tube may be adjusted by allowing some water to run out.†

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* For other methods, see Deschanel, Acoustics, p. 38.

† Very superior results can be obtained with apparatus of the form shown in Fig. 66. This form has the virtue of being complete within itself and the merit of being easy to construct.

Preliminary Discussion.—It may be shown that a wave motion travels a distance of one wave-length in the time occupied by one vibration of the particles participating in the motion. The distance which this wave motion travels in one second is called the velocity of the sound-wave.

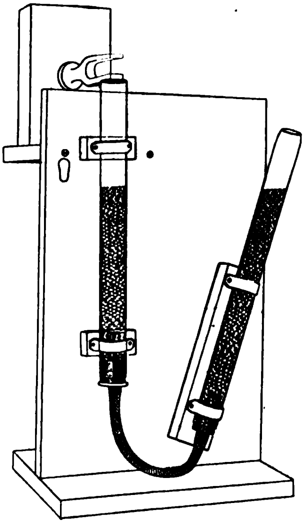

Fig. 66.

Whence, $V = lN$ (1)

where l is the wave-length and N is the vibration frequency.

Part A. — **Velocity determined from first resonance point.**

Directions.—(1) Hold the fork close to the mouth of the tube. Adjust the height of the water column in the tube until the fork produces the loudest tone. Immediately take the temperature of the room.

(2) Carefully measure the distance, d, between the lower prong and the water surface. The distance, d, equals $\frac{l}{4}$: Why? Measure the internal radius of the tube, r.

(3) Make several determinations of d and r. Take the mean of these measurements.

Calculations.—Compute the mean value of d and r. The diameter of the tube affects the wave-length. Rayleigh's correction on this account makes equation (1) read

$$V = 4\,(d + r)\,N \qquad (2)$$

where r is the radius of the tube, considered cylindriform, and d is the length of the air column.

Substitute in (2) and solve for V. The value of V thus found will be in error due to the fact that the velocity of sound-waves varies with the temperature. This velocity, in air, increases 0.616 M. for each centigrade degree. Apply this correction to determine the velocity at 0° C.

PART B.—**Velocity determined from other resonance points.**

PRELIMINARY STATEMENT.—There are other points where resonance effects can be produced, provided the tube is long enough. The tube must contain an air column of such length that the reflected wave will catch up with the vibrating prong as it passes the middle point of its arc of vibration on the *outward* motion.

DIRECTIONS.—(1) Find a second point of resonance as directed in (1) of Part A. Measure d and r as before. (Observe that d in this case is *not* $\frac{l}{4}$. What is it?)

(2) Find a third point of resonance, if the tube is long enough.

CALCULATIONS.—For any point of resonance the law given in equation (2) may be written in the form,

$$V = \frac{4(d+r)N}{2n-1}, \qquad (3)$$

in which n is the number of the resonance point.

Substitute in equation (3) and solve for V. Correct for temperature as directed in Part A.

EXERCISE 70

Velocity of Sound-Waves in Brass

APPARATUS.—A Kundt's apparatus, essentially as follows: A glass tube, about a meter long and of 3 or 4 cm. bore, is closed at one end with a cork. A brass rod, about 1.5 m. long, held by a clamp at its middle point, is provided with a piston which fits the glass tube loosely. The brass rod is passed into the tube until the piston is about 25 cm. from the closed end of the tube. Lycopodium is strewn within the tube.

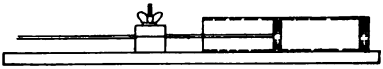

Fig. 67.

Preliminary Discussion.—When the brass rod is set in vibration, the air confined within the tube is thereby thrown into vibration. Evidently, whatever number of vibrations per second is made by the rod, the same number must be made by the air, since the vibration of the rod causes that of the air. That is,

$$N = n, \tag{1}$$

N being the frequency of the air and n that of the rod. Now, the velocity of sound-waves in air is given by the equation,

$$V = lN. \tag{2}$$

Also, the velocity of sound-waves in brass is given by a similar equation,

$$V_1 = l_1 n. \tag{3}$$

Dividing (2) by (3) we have,

$$\frac{V}{V_1} = \frac{lN}{l_1 n}. \tag{4}$$

Whence,

$$\frac{V}{V_1} = \frac{l}{l_1}.$$

Or,

$$V_1 = \frac{V l_1}{l}. \tag{5}$$

Directions.—(1) Rub the brass rod with a wet cloth, or a dry cloth sprinkled with rosin, the stroke passing from the middle toward the free end; a shrill, high note will be produced. Now, move the tube on its supports, or move the brass rod, until, when the rod is rubbed, the lycopodium is violently agitated and thrown into well-defined ridges with clear spaces between. Continue the adjustment until the end ridges are as perfect as the others.

(2) Measure the length, d, of the confined air column. Count the number of ridges, r.

(3) Measure the length, D, of the brass rod.

(4) Take the temperature of the air in the room.

(5) Repeat (1), (2), (3) and (4) several times. Take the mean of the several determinations.

THEORY AND CALCULATIONS.—By equation (5) we see that in order to compute the velocity of sound-waves in brass, we need only to know the wave-lengths in air and brass and the velocity in air. Consult a table for the velocity in air at 0° C. Then compute the velocity in air at the temperature observed during the performance of the exercise. Call this V of equation (5).

The ridges mark antinodal points, and the spaces between them mark nodal points. The distance between a node and an adjacent antinode is one-fourth of a wave-length.

Whence, $\frac{2d}{r} = l$, or one wave-length in air. (This is true for the reason that there are as many vibrating air columns in the tube as there are groups of ridges.)

Since the rod was clamped at its middle, that point must have been a node.

Whence, $2D = l_1$, or one wave-length in brass.

Substitute in equation (5) and solve for V_1. Compare result with the value given in tables.

EXERCISE 71

Laws of Vibrating Strings

APPARATUS.—A sonometer, held securely in a vertical position, provided with three piano-wires, A, B, and C. The diameter of A and B should be the same, while that of *C* should be exactly *twice* as great. Each wire should have a pail attached to its lower end to serve as a scale-pan. (Incline the sonometer slightly so that the wires will rest lightly against the lower head-piece.) A micrometer caliper. A meter-stick. A balance and set of weights. A tuning-fork of pitch C-128.

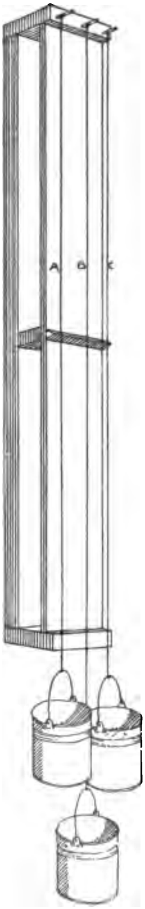

FIG. 68.

PRELIMINARY STATEMENTS. — The object of this exercise is twofold:

(I) To familiarize the student with certain general ideas concerning vibrating strings.

(II) To verify the following laws of vibrating strings:

(1) The frequency of a vibrating string varies *inversely* as its length. (2) The frequency of a vibrating string varies *inversely* as its diameter. (3) The frequency of a vibrating string varies *directly* as the square root of the tension applied to the string.

NOTE.—In (1) the diameter and tension are assumed to be constant; in (2) the length and tension are constant; and in (3) the length and diameter are constant.*

In the work of this exercise one string is to be tuned to the pitch of the fork, C-128. This has to be done for the reason that the average high-school laboratory is not provided with apparatus for determining vibration frequencies by a direct method. Another string is to be tuned to the octave above C-128. The student will have to assume as proved that the pitch of the latter string is C-256. Thus, he will know the vibration frequencies of these strings at the outset. Call the frequency of the former N and that of the latter N_1.

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* Do not fail to include in your report on Part B a definite statement showing how these conditions were fulfilled.

Part A.—**General ideas regarding the relation of length, diameter, and tension to vibration frequency.**

Directions.—(1) Place weights in the scale-pans of *A* and *B* until they both have the pitch of the fork, C-128. Place a bridge under *B* at any point other than the middle. Pluck *A* and the longer segment of *B*. Which one gives a note of higher pitch? Place the bridge at other points under *B*. Note the relative pitch as before. Place the bridge under *B* at its middle point. Pluck either segment of *B*. Does it give the octave above *A*? If it does, this will show that the frequency is doubled if the length of the string is halved.

(2) Place *equal* weights in the scale-pans of *B* and *C*. Pluck both wires. Which one has the higher pitch?

(3) Place *unequal* weights in the scale-pans of *A* and *B*. Pluck both wires. Which one gives a note of higher pitch? *Double* the weight in *B* and note whether the string gives the octave above or below the note given by *A*. If it does not, this will show that the frequency does not vary with the tension in a simple way.

Part B.—**To verify the laws of vibrating strings given above.**

Directions.—*Law of Length.*

(1) Place weights in the scale-pan of *A* until the string vibrates in unison with the fork, C-128. Note the total weight, including that of the scale-pan. Now place weights in the scale-pan of *B* until the total weight suspended from *B* is the same as that suspended from *A*.

(2) Place a bridge under *B* and move it along until one segment of the string gives the octave above C-128. Measure the length of *A* and the segment of *B* just mentioned.

Law of Diameter.

(1) Place weights in the scale-pan of *C* until the string vibrates in unison with the fork, C-128. Note the total weight as directed in (1) above. Make the total weight suspended from *B* equal to the total weight suspended from *C*.

(2) Pluck *B*, and note that its pitch is about an octave above C-128. If such is *not* found to be the case, either the tensions are not equal or the diameter of *C* is not twice that of *B*. (If a well-made sonometer is used it may be assumed that the lengths are equal.)

Law of Tension.

(1) Place weights in the scale-pan of *A* until the string vibrates in unison with the fork, C-128.

(2) Place weights in the scale-pan of *B* until the pitch of the string is an octave above that of *A*. Note the total weights suspended by both strings.

CALCULATIONS AND RESULTS.—Express the laws of length, diameter, and tension in the form of proportions. Substitute proper numerical results in each proportion and verify.

Combine the three proportions you have framed into a single proportion.

Include in your report a brief discussion of the application of the laws of vibrating strings to such instruments as the piano, the violin, the harp, etc.

EXERCISE 72

Vibration Ratios of the Diatonic Major Scale

APPARATUS.—A sonometer provided with but one string. A zither or other instrument which may be used to give the successive tones of the scale under consideration.

PRELIMINARY DISCUSSION.—In Exercise 71 it was shown that when the diameter and tension are constant, the vibration frequency of a string varies inversely as the length of the string. It follows that we can determine the vibration ratios by measuring the length of the strings which produce the notes of the scale without determining the vibration numbers themselves.

Directions.—(1) Tune the zither, using C-128 as its lowest note. Tune one string of the sonometer to the note, C-128, given by the zither. Measure the length of this string.

(2) Adjust the bridge under the string until the segment is in unison with the note D of the zither. Measure the length of the segment as before. Proceed in a similar manner with E, F, G, and so forth.

Record data in the following form:

Note.	Length in cm.	Computed vibration ratio.	True ratio.	Computed vibration number	True vibration number.
C			1	(128)	128
D					
E					
F					
G					
A					
B					
C^1					

Calculations.—Calling the lengths of the C and D strings L and L_1, and their vibration numbers N and N_1, respectively, we have, by Exercise 71,

$$N : N_1 :: L_1 : L.$$

But $\frac{N}{N_1}$ is the vibration ratio of C to D.

Whence the vibration ratio of C to D is $\frac{L_1}{L}$. In the same manner, compute the vibration ratio of each note to C.

CHAPTER VIII

LIGHT

EXERCISE 73

Intensity of Illumination: Law of Inverse Squares

APPARATUS.—Three screens, *A*, *B*, and *C*, of different sizes, *A* being the smallest and *C* the largest. A candle. An optical bench, or, in absence of one, the candle and screens may be arranged on the work-table, and distances measured with a meter-stick.

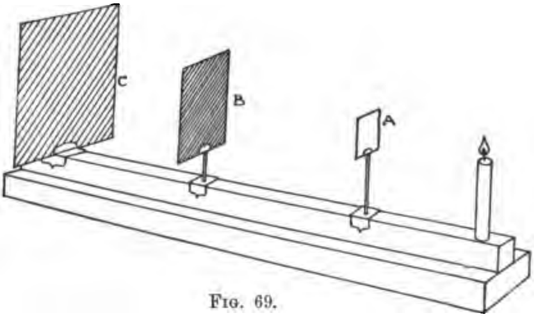

FIG. 69.

PRELIMINARY DISCUSSION.—The meaning of the law of inverse squares may be illustrated by taking the case of two equal areas, *E* and *F*, situated at unequal distances from the same source of light, *F* being twice as far therefrom as *E*. Now, the light intensity on area *F* will, of course, be less than that on area *E*. Furthermore, it may be shown, both experimentally and by geometry, that these intensities and the distances bear the following relation to each other:

$$I : I_1 : : d_1^2 : d^2,$$

in which I and I_1 are the light intensities on E and E, respectively, and d and d_1 are their distances, respectively, from the common light source. That is, at E, which is only one half as far from the light as F, the light intensity will be four times that at F.

DIRECTIONS.—(1) Adjust the position of the candle until the center of its flame has the same height as the center of the screens.

(2) Place the screen A about 10 cm. from the candle. Place screen B at such a distance behind A that it shall be exactly covered by the shadow cast by A.

(3) Place C behind B at such a distance that the common shadow cast by A and B shall exactly cover it.

(4) Measure D, D_1, and D_2 the distances, respectively of A, B, and C, from the candle flame. Measure the area of A, B, and C, calling that of each a, a_1, and a_2, respectively.

(5) Repeat (1), (2), (3), and (4), placing A at a distance of 20 cm. from the candle.

CALCULATIONS.—Let the intensity of light on A, B, and C be represented, respectively, by I, I_1, and I_2. Then I_1, the intensity of light on B, will be $\frac{a}{a_1}$, since the light which falls on a unit of surface of A will, on the removal of A, be distributed over a_1 units of surface of B. Likewise, the intensity I_2, on C, will be $\frac{a}{a_2}$.

Compute I_1 and I_2. Substitute in the equations which follow, and verify.*

$$I : I_1 :: D_1^2 : D^2, \qquad (1)$$

$$I : I_2 :: D_2^2 : D^2. \qquad (2)$$

QUERIES.—What geometrical reason can be given for the law of inverse squares? Suppose a certain electric light, at a distance of 1 foot, to have an intensity equal to that of 2,000 candles at the same distance; if the electric light were at a distance of 40 feet, the intensity of the light received from it would equal that of how many candles at a distance of 1 foot?

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* In verifying (1) and (2), let I be represented by 1.

EXERCISE 74

Candle-Power of a Gas-Jet and an Incandescent Electric Lamp by means of a Bunsen Photometer

APPARATUS.—A Bunsen photometer, consisting essentially of a screen having a translucent spot at or near its center. This screen may be mounted on an optical bench, or, in the absence of one, the table-top and a meter-stick may serve instead. A straight sperm candle cut into two equal parts. Lamps to be tested.

PRELIMINARY DISCUSSION.—The light-giving power of illuminants is measured in terms of a unit known as the standard candle-power. (What is a standard candle?) Do not fail to bear in mind that when we speak, for example, of a 16 c. p. lamp, we attach to the number 16 the following significance, and no more. A 16 c. p. lamp *should* give, at a stated distance, sixteen times as much light as a standard candle gives at the same distance.

Nearly all methods used in photometry are based upon the principle of inverse squares. The illuminant under measurement is usually placed at such a distance from a screen that the intensity of its illumination of the screen shall equal that of a *standard candle.* When these intensities are the same, it can be shown that the following equation is true:

$$c : c_1 :: d^2 : d_1^2,$$

where c and c_1 are the candle-powers, and d and d_1 are the distances, respectively, of the standard candle and the given illuminant from the screen.

The correctness of this equation will be apparent after a consideration of the following argument: Suppose a standard candle to be placed 1 foot from a screen. Suppose, further, that a gas-jet, placed 5 feet from the screen on the opposite side, illuminates the screen with an intensity equal to that of the standard candle. Now, by the law of inverse squares, if the gas-jet is located 1 foot from the screen, the intensity of its illumination of the screen will be 25 times what it was at a distance of 5 feet. But, by the definition of candle-power given above, its intensity of illumination at a distance of 5 feet from the screen was 1 c. p. Hence, the actual candle-power of the jet is 25.

DIRECTIONS.—(1) Place the two half-candles close together, side by side. Place the candles about 1 foot from the screen. The candles should be allowed to burn for several minutes before the test is begun. The proper condition of the candles is shown when the tips of the wicks curl over slightly and glow. Adjust the candles so that the plane of one curl shall be perpendicular to that of the other.

(2) Now, move the screen to and fro to find a position such that the translucent spot shall be equally illuminated when viewed from either side. This, however, will only be the approximate position of the screen. To find the correct position, proceed as directed in (3).

(3) Place the screen too far toward the right. Then move it gradually toward the left until both sides of the screen appear equally illuminated. Mark this position. Now place the screen too far toward the left, and then move it gradually toward the right until equal illumination of both sides of the screen is produced, as before. Mark the second position of the screen. A point half-way between these two points will be the correct position of the screen.

(4) Measure the distance, d, from the correct position of the screen to the candles, and d_1, that from the jet to the screen.

(5) Repeat (2), (3), and (4).

(6) Repeat (2), (3), and (4), to test the candle-power of an incandescent electric lamp.

(7) Repeat (6).

Record data in a neat form.

CALCULATIONS.—Compute mean values of d and d_1 for each case. Substitute in the above equation and solve for the candle-power of each illuminant. (Assume the illuminating power of the two half-candles to be 2 c. p.)

EXERCISE 75

Law of Reflection

APPARATUS.—A drawing-board. A plane mirror held in a vertical position. A 10-cm. protractor. A short rule or straight-edge.

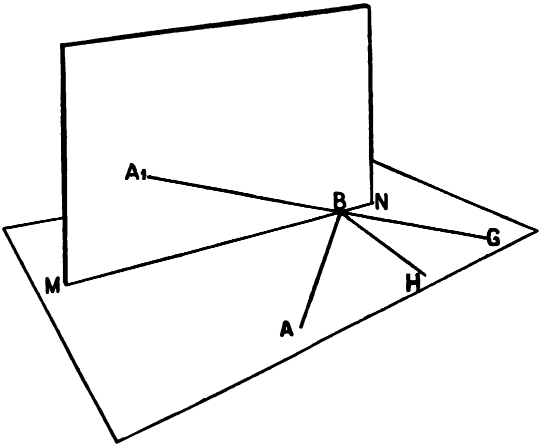

FIG. 70.

DIRECTIONS.—(1) Draw a straight line, MN, on a sheet of paper to represent the reflecting surface of the mirror. Draw a line, AB, in an oblique direction to MN, B being the point of intersection of AB with MN.

(2) Place the back edge of the mirror on the line MN. Place the straight-edge in such a position that, in sighting along its edge, the latter and the image of the line AB, *as seen in the mirror*, shall be in the same straight line. Without moving the straight-edge, draw the line BG in the same straight line with the image of AB.

(3) From the point B let fall BH, perpendicular to MN. Draw this perpendicular on the anterior side of MN.

(4) Which, ABH or GBH, is the angle of incidence? Measure the angles of incidence and reflection with the protractor.

(5) Repeat (1), (2), (3), (4), several times, recording all data in a neat form.

EXERCISE 76

To determine by Construction the Character of an Image formed by a Plane Mirror

APPARATUS.—A drawing-board. A large sheet of paper. A short rule. A protractor. Very short pins.

PRELIMINARY DISCUSSION.—The image of any object consists of an indefinite number of minute, overlapping images. Therefore, in determining the character of an image *by construction*, we must confine ourselves to a few limiting points.

In order to make the case as general as possible, let it be required to construct the image of three points in a regularly curved line whose extremities are situated at unequal distances from the mirror.

The construction of this image should establish four principles with respect to the image of a given point:

(1) Plane mirrors form virtual images only.

(2) The image of the point is in the line of light which enters the eye.

(3) The image of the point appears as far behind the mirror as the point is in front of it.

(4) A straight line drawn from the image to the point is perpendicular to the mirror.

Since the image of any object is merely an assemblage of images, the foregoing principles apply to the image of an object as well.

DIRECTIONS.—(1) Draw MN to represent the reflecting surface of the mirror. At a short distance in front of MN draw a regularly curved line, ABC, whose image is to be constructed. Locate B at the middle point of the curve, and the extremities of the curve at unequal distances from MN. Stick a pin at A. At a distance of at least 10 cm. in front of MN locate three widely separated positions of the eye, E, E_1, and E_2, lettered in this order from left to right. Stick pins at E, E_1, and E_2.

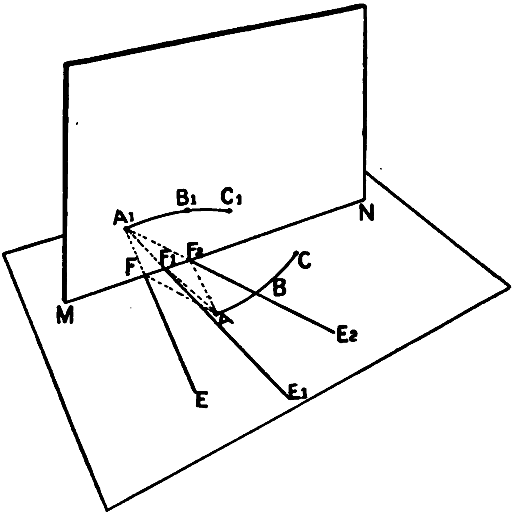

FIG. 71.

(2) Now place the edge of the reflecting surface of the

mirror on the line MN. Support the mirror firmly in a vertical position.

(3) Place one edge of the rule so that it shall have the direction of a straight line joining E and A_1, the image of A as seen in the mirror, and draw a straight line from E to the mirror. In the same manner, locate and draw straight lines from E_1 and E_2 respectively, toward A_1. These three lines will intersect MN at different points. Call these points F, F_1, and F_2 respectively, lettered from left to right. Remove the mirror temporarily, and draw the dotted lines AF, AF_1, and AF_2. Replace the mirror. See that it occupies the precise position it did before. Now, looking into the mirror, the student will observe that the images of AF, AF_1, and AF_2 appear to be the respective continuations of the straight lines EF, E_1F_1, and E_2F_2, intersecting behind the mirror at the common point A_1.

(4) Proceeding as directed in (3), locate and draw straight lines toward B_1, the image of B, from E, E_1, and E_2 respectively, to the mirror.*

(5) In the same manner locate and draw straight lines toward C_1, the image of C, from E, E_1, and E_2 respectively, to the mirror.

(6) Remove the mirror and produce the lines, drawn as directed in (4), to their intersection at B_1, back of MN.

(7) Produce the lines referred to in (5) to their intersection at C_1.

(8) Connect the points A_1, B_1, C_1 with a smooth curve. The line thus found is the image of the line ABC.

(9) At the points F, F_1, F_2 erect perpendiculars to, and in front of, MN. Letter these perpendiculars FG, F_1G_1, and F_2G_2, respectively.

(10) From the point A drop a perpendicular to MN

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* Before proceeding with this operation, remove the pin from A to B. It is confusing to have more than one pin at a time near the mirror.

and produce it to A_1. Let L be the point of intersection of AA_1 with MN.

(1) Measure the angles of incidence and reflection for each of the three rays which originate at A.

RESULTS AND CONCLUSIONS.—Include in your report an extended discussion of how the work of this exercise verifies each of the principles laid down in the preliminary discussion.

Prove, by geometry, that AA_1 is perpendicular to MN at L and that $AL = A_1L$. Explain facial inversion of the image.

Name, by letters, the three incident rays which originate at A. Name, by letters, the three reflected rays which determine the position of the image of A. Name, by letters, the angles of incidence and reflection formed by the three rays which originate at A.

EXERCISE 77

To determine by Construction the Character of an Image formed by a Concave Cylindrical Mirror

APPARATUS.—A drawing-board. A large sheet of paper. A short rule. A concave cylindrical mirror.* A concave spherical mirror. Short pins.

PRELIMINARY DISCUSSION.—The importance which attaches to a thorough understanding of cylindrical mirrors lies in the fact that, in a theoretical way, the spherical mirror may be developed from the cylindrical mirror. Before attempting this exercise the student should have correct notions touching the following points:

1. How does a concave cylindrical mirror differ from a concave spherical mirror?

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* It is not necessary to provide expensive mirrors for this and the following exercise. Pieces of cylindrical mirrors, cut to the size 4 x 6 inches, can be procured at small cost of makers of mirrors.

2. How may the concave spherical mirror be developed, geometrically, from the concave cylindrical mirror?

3. How do images of the same object formed by cylindrical and spherical mirrors differ from each other?

The student will discover as he proceeds that the method of the exercise is to construct that portion of the whole image of a given object which is produced by a horizontal section of the mirror.

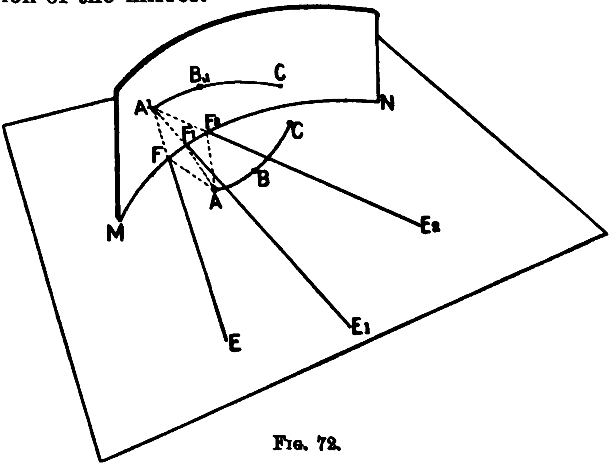

Fig. 72.

Directions.—(1) Place the mirror in the middle of the sheet of paper and so that its plane aspect shall be in the vertical. Draw a line, MN, having the curvature of the mirror. Support the mirror firmly in a vertical position, with the lower edge of its reflecting surface on this line. At a short distance in front of MN draw an arc of a circle of small radius. Letter this arc ABC, as in Exercise 76. (The extremities of this arc should be at unequal distances from MN. See Preliminary, Exercise 76.) Stick a pin at A.

(2) Locate three positions of the eye, E, E_1, and E_2, as directed in Exercise 76, and let the images of the points A, B and C be known as A_1, B_1, and C_1, respectively. Stick pins at E, E_1, and E_2.

(3) Proceed as directed in (3), (4), and (5) of Exercise 76 to locate A_1, B_1, and C_1, the images, respectively, of A, B, and C. At the point F erect a perpendicular to the mirror, FG.

(4) Trace the curve of the mirror on another sheet of paper. Determine the distance from the vertex to the center of curvature of the mirror by erecting perpendiculars to the curve at two points. The point of intersection of these perpendiculars is the center of curvature. Locate the principal focus.

(5) Hold a pencil in a vertical position beyond the center of curvature, between the center of curvature and the principal focus, and between the principal focus and the vertex. Observe, in each position, the character of the image as to (1) size, (2) position (erect or inverted), (3) location, (4) lateral and vertical dimensions as compared with those of the object, and (5) kind (whether real or virtual). Observe the character of these images both in the vertical and in the horizontal.

RESULTS AND CONCLUSIONS.—Include in your report a tabulated statement which shall define the character of an image formed by a concave cylindrical mirror, covering the points mentioned in (5). Fasten in your note-book the sheet of paper on which the construction called for in the exercise was made.

Describe the path of the light ray which came from A to the eye, in each of the three positions, E, E_1, and E_2. Name, by letters, the incident rays which originate at A and the reflected rays to which they give rise. Name, by letters, the angles of incidence and reflection for the case of incident ray AF.

If a concave cylindrical mirror is held so as to be plane in the vertical and an object is located between the principal focus and the vertex of the mirror, the image will appear to be short and very thick. If the mirror is held so as to be plane in the horizontal, and the object remains in the same position, the image will appear to be very long and slender. Explain fully how these effects are produced.

Under the first of the two conditions just given, how does the size of the image, *in the vertical*, compare with that of the object in the vertical?

Under the second condition, how does the size of the image, *in the horizontal*, compare with the size of the object in the horizontal? Explain fully.

EXERCISE 78

To determine by Construction the Character of an Image formed by a Convex Cylindrical Mirror

APPARATUS.—The same as that for Exercise 77, except that a convex mirror is substituted for the concave mirror.

DIRECTIONS.—(1) Proceed precisely as directed in (1), (2), and (3) of Exercise 77.

(2) Hold a pencil in a vertical position at different distances from the mirror. Note whether there is any distortion of the image, as was the case in the preceding exercise. Compare the size of the image when the object is near the mirror with the size of the image when the object is farther away from the mirror.

(3) Discover whether there are any *critical* positions of the object—i. e., positions so located that if the object be removed beyond them, or placed between them and the mirror, the character of the image, with respect to whether it is real or virtual, erect or inverted, etc., will change in the manner observed in the performance of Exercise 77.

CONCLUSIONS.—Include in your report a full discussion of the character of the image formed by a convex cylindrical mirror. Cover, in full, the points indicated in Results and Conclusions of Exercise 77, in so far as they are applicable to this exercise.

QUERIES.—Suppose the mirror you used had the same degree of curvature in all its sections that it now has in the section which is perpendicular to the plane aspect of the mirror, what sort of a mirror would it then be?

What effect does distance have upon the image formed by a convex cylindrical mirror? Give reasons for your answer which are based upon the laws of light.

EXERCISE 79

A Study of the Character of Images produced by Concave and Convex Spherical Mirrors

APPARATUS.—Two spherical mirrors of small aperture, one concave and the other convex. A screen. A candle.

DIRECTIONS.—(1) Place the lighted candle in each of the objective positions specified below. Focus the image of the flame on the screen. Observe the character of the image when the object (the candle) is in each of the following positions:

1. Beyond the center of curvature.
2. At the center of curvature.
3. Between the center of curvature and the principal focus.
4. At the principal focus.
5. Between the principal focus and the vertex.

(2) Place the lighted candle at different distances from the convex mirror. Discover whether there are any critical positions for this mirror. Observe the character of the image for different positions.

Record observations in a tabular form.

RESULTS.—The tabular summary should include the principal points in the exercise as defined in direction (5) of Exercise 77.

What would be the effect upon the image if the mirror

were rotated upon its principal axis? Give reasons for your answer.

Explain why convex mirrors always produce reduced images.

EXERCISE 80

Focal Length and Law of a Convex Lens for the Case of a Real Image

Apparatus.—An optical bench. A convex lens. An opaque screen to be known as No. 1. Another opaque screen having fine cross-wires stretched across an opening in the screen. Call the latter screen No. 2.

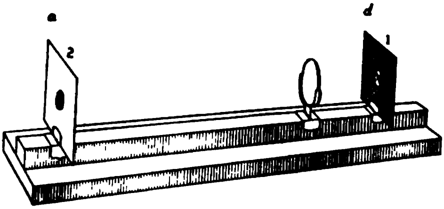

Fig. 73.

Preliminary Discussion.—The focal length of a lens is the distance from the optical center of the lens to its principal focus. Letting f be the principal focal length of a given convex lens, and P and p be the distances, respectively, of the object and image from the optical center, the relation which exists among f, P, and p is expressed by the equation

$$\frac{1}{f}=\frac{1}{P}+\frac{1}{p}.$$

Part A.—**To determine the focal length of the lens.**

Directions.—(1) Place the bench in such a position that it shall point through an open window toward a distant object, or (which will answer fairly well) toward an incandescent lamp placed at the farther end of the room.

(2) Move the screen back and forth until the image of the object used is sharply focused upon the screen. Meas-

ure *f*, the distance from the screen to the optical center of the lens.*

Part B.—**To prove the law of the convex lens.**

(1) Place screen No. 2 at one end of the optical bench. Just back of this screen place a lighted candle. Place screen No. 1 at the other end of the bench. Place the lens between the two screens.

(2) Now move the lens back and forth until a sharp image of the cross-wires is focused upon screen No. 1. Measure the distance P, from the cross-wires to the optical center of the lens, and p, the distance of screen No. 1, from the optical center.

(3) Place screen No. 2 in other positions, and repeat (1) and (2).

Record data in a neat form.

Calculations.— Substitute in the equation given above, and verify.

Queries.—What is meant by the expressions: A lens of long focal length? A lens of short focal length? A two-inch lens?

EXERCISE 81

Magnifying Power of a Single Convex Lens

Apparatus.—An optical bench. A convex lens. A screen ruled to millimeters. A transparent screen on which is ruled a short horizontal line to be used as the object.

Preliminary Discussion.—The simplest case of magnification is that produced by a single convex lens. When an object is viewed through such a lens, the lens becomes a magnifying-glass, or a simple microscope, as it is sometimes called. (Where must the object be placed with respect to the lens in order that a convex lens may be used as a magnifier?) Calling a given diameter of an object AB, and that of the corresponding diameter of the image A_1B_1, the magnifying power of a lens is given by the equation,

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* For a definition of the optical center, consult a text-book on general physics or a treatise on light.

$$M = \frac{A_1B_1}{AB}. \tag{1}$$

The same result may be reached in a different manner. The apparent size of an object is determined by the ratio of the visual angle subtended by the object to that of the visual angle subtended by the image; or,

$$M = \frac{\text{Visual angle of image}}{\text{Visual angle of object}} = \frac{\angle i}{\angle o}. \tag{2}$$

But it may be shown that, for small angles, $\angle i$ is proportional to the distance of the image from the eye, and $\angle o$ to the distance of the object from the eye. Now, in order to have easy and distinct vision, a normal eye views the object at a distance of about 25 cm., this quantity being known as the *least distance of distinct vision*, d. When a magnifying-glass is used, it is assumed that the relative positions of object and lens are adjusted so as to cause the image to subtend the same angle that an object of the same size would subtend at the least distance of distinct vision. Hence, A_1B_1 is proportional to d, or to 25 cm.

Finally, it may be shown that in such cases the object is placed quite close to the principal focus of the lens. The visual angle of the object at this distance is proportional to the principal focal length, for the reason already given. Hence, AB is proportional to the focal length, f.

That is,

$$\frac{A_1B_1}{AB} = \frac{d}{f}. \tag{3}$$

Whence,

$$M = \frac{d}{f}. \tag{4}$$

DIRECTIONS.—(1) Place the millimeter screen at the least distance of distinct vision from the anterior surface of the lens (the eye is supposed to be quite close to the lens). Place the transparent screen near the principal focus of the lens.

(2) Now, looking through the lens at the mark with one eye, and at the millimeter screen with the other, move the object, i. e. the transparent screen, back and forth until a clear image is seen against the millimeter screen as a background. Adjust the position of the transparent screen laterally until the left-hand end of the mark coincides with a numbered vertical line on the millimeter screen.

(3) Observe carefully the millimeter mark to which the image appears to extend toward the right hand. From these two observations determine the apparent length, A_1B_1, of the image. Count the number of scale divisions over which the object actually extends. This length is AB.

(4) Repeat (2) and (3) several times.

(5) Determine the principal focal length of the lens as directed in (1) and (2) of Exercise 80. Calculate mean values of AB and A_1B_1.

CALCULATIONS.—Substitute in equations (1) and (4) and solve for M. Compare results.

QUERIES.—Which expression is to be preferred, a magnification of 25 *times*, or, a magnification of 25 *diameters*? Give reasons for your answer. Give the meaning of each expression.

As a matter of fact, does the image of a certain small object, as seen through a magnifying-glass, cover more of the retina than when seen unaided by a glass? If such is actually the fact, positive magnification must always be at the expense of what?

EXERCISE 82

A Study of a Combination of Two Convex Lenses and a Determination of the Magnifying Power of a Simple Refracting Telescope

APPARATUS.—An optical bench. Two convex lenses, one of long and the other of short focal length. The two screens used in Exercise 80. A candle.

PART A.—**Study of a combination of two convex lenses.**

PRELIMINARY DISCUSSION.—The two lenses specified above may be so placed with respect to each other as to produce:

(1) A real image—the lenses as used to produce this effect constituting no optical instrument of value.

(2) A virtual image—the lenses as used to produce this effect constituting a simple telescope or microscope, depending upon the position which each occupies with reference to the other, and upon their comparative focal lengths.

DIRECTIONS.—(1) Place a lighted candle behind the illuminated cross-wires which serve as an object. Place both lenses on the bench, the one of greater focal length nearer the object, so that the distance between their optical centers shall be *slightly more* than the sum of their focal lengths.

(2) Place the other screen on the bench behind the lens of shorter focal length, and move the screen back and forth until a sharp image of the cross-wires is focused upon it. (What kind of an image is this?)

(3) Without disturbing the apparatus in any way, hold a piece of cardboard between the two lenses in the proper position to catch thereon a sharp image of the cross-wires. (Bear in mind that the image is there, whether or not the screen is in place to receive it.) Compare the character of this image with that of the image formed by both lenses.

(4) Without disturbing the cross-wires or the other

screen, move the eye-lens toward the object-lens until their distance apart is *slightly less* than the sum of their focal lengths.

(5) Determine whether it is now possible to catch an image of the cross-wires on the back screen. Point the bench toward a lighted lamp situated ten or fifteen meters away. Determine whether it is possible to catch an image of the lamp on the back screen.

(6) With the eye close to the eye-lens, look through this improvised telescope at a lighted lamp placed at the farther end of the room. Move the eye-lens back and forth to secure a perfect focus of the image on the retina. Note the apparent position of the image as now seen in order to be able to answer as to the character of this image. Compare the size of this apparent image with that of the real image of the lamp produced by the object-lens and eye-lens.

CONCLUSIONS.—Include in your report on Part A a summary of the more important facts and deductions to which this part of the exercise leads.

PART B.—**To determine the magnifying power of the combination used as a telescope.**

PRELIMINARY DISCUSSION.—A general and simple equation for the magnifying power of a refracting telescope may have the form of equation (1) of Exercise 81; or,

$$M = \frac{A_1B_1}{AB}. \qquad (1)$$

In this exercise, as was the case in Exercise 81, it is desirable to deduce an equation easier of application than the one given in (1).

In doing this it should be borne in mind that *both* lenses produce magnification. The character of these magnifications, and their combined effect, may be made clearer by the following:

Consider, first, the real image produced by the object-lens. Viewed from the optical center of the object-lens, this image and the object will appear to have the same size. (Why?) If, however, the position of the eye be changed so that the distance, d, from it to the image is the *least distance of distinct vision*, or 25 cm., the apparent size of the image will be considerably increased, while the apparent size of the object will not be materially decreased, since the apparent distance of the object, represented by the focal distance, F, remains unchanged.

Remembering that (when the angles are comparatively small) the visual angle varies inversely as the distance, it follows that

$$\frac{\angle i}{\angle o} = \frac{F}{d} = \frac{F}{25}.$$

Consequently, the magnifying power of the object-lens is $\frac{F}{25}$. (Compare this statement with equation (4) of Exercise 81.)

Now, the real image produced by the object-lens is viewed at a distance from the eye-lens a trifle less than the focal length of the latter. The eye-lens, therefore, becomes a magnifying-glass which acts upon the real image. That is, we are now dealing with a magnification of a magnification. But as the magnifying power, M, of a simple microscope has already been shown to be $\frac{25}{f}$, it follows that the combined magnification produced by both lenses is given by the equation,

$$M = \frac{25}{f} \times \frac{F}{25}.$$

$$\text{Whence, } M = \frac{F}{f}. \qquad (2)$$

Directions.—(1) Measure the focal length, F, of the object-lens, and f, that of the eye-lens.

(2) Point the optical bench toward an illuminated screen located at the farther end of the laboratory.

Against this screen, as a background, is placed a lighted incandescent electric lamp. Place the lenses on the bench, the object-lens nearer the lamp, so that the distance between them shall be a trifle less than the sum of their focal lengths. Place a scale close to the lamp, in a vertical position.

(3) Focus this improvised telescope upon the lamp. Now, with one eye looking through the telescope, while the other is looking directly at the screen, do two things: (1) Count the number of scale divisions over which the image of the lamp appears to extend. (Or, a fellow student may locate two points on the scale, the upper one on a level with the upper end of the filament of the lamp, and the lower one on a level with the lower end of the filament. The distance between these points will be the apparent size of the image.) Call this apparent length A_1B_1. (2) Count the divisions of the scale over which the filament actually extends. Call this length AB.

CALCULATIONS.—Substitute in equations (1) and (2) and solve for M. Compare results.

QUERIES.—Magnification is produced at the expense of what? Why is an objective of large diameter a desirable thing in the case of a telescope? (In compound microscopes the great loss of light occasioned by the small objectives which must be used to obtain high powers is compensated for by condensers that flood the object with light.)

EXERCISE 83

The Refracting Angle of a Prism

APPARATUS. — A drawing-board. A large sheet of paper. A prism. A protractor. A short rule.

PRELIMINARY DEFINITION.—The refracting angle of a prism is the angle opposite the side around which a ray of light is bent in passing through a prism.

DIRECTIONS.—(1) Fasten the sheet of paper to the drawing-board with thumb-tacks. Draw two parallel lines, *EF* and *GH*, about a centimeter apart, from the middle of the sheet nearly to its margin. Stick two pins in each line in a vertical position. Place the prism with the refracting angle *A* between the lines. Call the base angles of the prism *B* and *C*.

(2) Place the rule in such a position at an oblique angle to the side of the prism, *AC*, that one edge shall be directly in line with the reflected ray which is produced by the incident ray, *GH* (or, what amounts to the same thing, in line with the images of the pins). Draw a line *HL* to represent the reflected ray.

(3) In the same manner, locate and draw *FK*, the reflected ray from the side of the prism, *AB*.

(4) Produce *FK* and *HL* until they intersect at the point *O* within the angle *BAC*. Measure the angle *KOL* with a protractor.

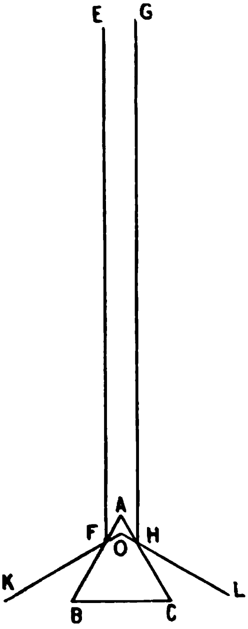

FIG. 74.

(5) Repeat (1), (2), (3), and (4) several times.

CALCULATIONS.—Compute the mean value of the angle *KOL*. This method of determining the refracting angle *A* depends upon the simple geometrical proposition,

$$\angle BAC = \angle \frac{KOL}{2}.$$

Include a demonstration of this proposition in your report.

EXERCISE 84

Determination of the Index of Refraction by the Method of Minimum Deviation

Apparatus.—A glass prism whose refracting angle is known. A drawing-board. A large sheet of paper. A protractor.

Preliminary Discussion.—Let ABC represent a section of the prism parallel to its base, A being the refracting angle. Let a ray of light from the point S be incident upon AB at E. Let the ray emerge from the prism at F in the side AC and thereafter have the direction FH. Let GE be perpendicular to AB at the point E, and KF be perpendicular to AC at the point F. Produce SE and FH to their intersection at d, and GE and KF to their intersection at O. Let D be the exterior angle opposite to the angles dEF and dFE. Assume the refracted ray, EF, to be parallel to BC.

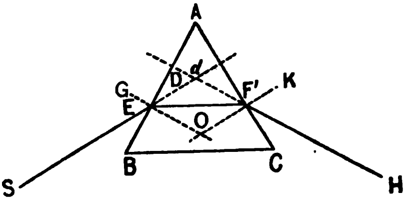

Fig. 75.

Stated in its simplest form, the index of refraction, I, is given by the equation,

$$I = \frac{\text{sine of angle of incidence}}{\text{sine of angle of refraction}} = \frac{\sin \angle \, GES}{\sin \angle \, FEO}. \quad (1)$$

In actual practice the angles of incidence and refraction are not measured directly. Instead, the index is determined from the refracting angle (already measured) and from the angle of minimum deviation, D. (For methods of measuring this angle, consult the authorities cited below.) *

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* Loudon & McLennon, p. 54. Glazebrook's Heat and Light, p. 53.

Now, when the index is measured by the method of minimum deviation, it may be shown that

$$\frac{\angle GES}{\angle FEO} = \frac{\frac{1}{2}(\angle D + \angle A)}{\frac{1}{2}\angle A}. \qquad (2)$$

$$\text{But, } I = \frac{\sin \angle GES}{\sin \angle FEO}. \qquad (3)$$

$$\text{Whence, } I = \frac{\sin \frac{1}{2}(D + A)}{\sin \frac{1}{2} A}. \qquad (4)$$

DIRECTIONS.—(1) Fasten a large sheet of paper to the drawing-board. Draw a straight line, *SE*, to represent an incident ray from a point in the lower part of the left-hand margin to a point near the center of the sheet of paper. Place the prism on the paper so that *SE* shall intersect the side, *AB*, at an oblique angle. Now, with the eye at *H*, a point having about the same angular position relative to *AC* as *S* has with respect to *AB*, the incident ray, *SE*, will appear to have the direction of *FH*, produced. With the eye still at *H*, rotate the prism through a small angle about its vertical axis. This will cause the apparent incident ray to change its position somewhat. Find a position of the prism such that a small rotation of it will not cause the apparent incident ray to change position. This position will produce a minimum deviation of the ray in passing through the prism.

(2) Being very careful not to disturb the prism in the least, draw the emergent ray, *FH*, as nearly as possible in line with the image of the incident ray, *SE*. Trace the sides of the prism with a sharp pencil.

(3) Remove the prism and complete the figure as shown above. Measure *D*, the angle of minimum deviation, with great care. Also measure the angle of incidence, *GES*, and the angle of refraction, *FEO*.

CALCULATIONS.—Find the sines of these angles in a table of sines. Substitute in equation (1) and solve for *I*. Substitute in equation (4) and solve for *I*. Compare results.

APPENDIX

Metric Units and Equivalents

UNITS.—The units which are used most in physics are the meter, the kilogram, and, occasionally, the liter, with their multiples and submultiples. The metric system is easy in the extreme if one but remembers the meaning of the prefixes employed. The principal ones follow:

Milli, meaning .001 of the unit.
Centi, " .01 "
Deci, " .1 "
(The unit.)
Deka, meaning 10 of the unit.
Hekta, " 100 "
Kilo, " 1000 "

EQUIVALENTS.—Except in rare cases, the following equivalents will be sufficient for all necessary conversions from English to metric, or from metric to English, units:

Length

1 m. = 39.37 in.
1 cm. = 0.3937 in.
1 in. = 2.54 cm.

Volume

1 = 1,000 cc. = 1.0567 qts.

Weight

1 kg. = 1,000 g. = 2.2046 lbs.
1 lb. = 453.59 g.

Miscellaneous

1 cc. of pure water at 4° C. weighs 1 g.

1 cu. ft. of pure water at 4° C. weighs 62.425 lbs.

1 cu. ft. of pure water at 16°.66 C. weighs 62.231 lbs.

1 cu. ft. of pure air at 0° C., pressure 76 cm. mercury, weighs 0.080728 lb.

1 cu. ft. of pure hydrogen, same temperature and pressure, weighs .00594 lb.

1 cc. of pure dry air, same temperature and pressure, weighs .0012932 g.

1 cc. of pure hydrogen, same temperature and pressure, weighs .0000896 g.

Numbers and Formulæ frequently Used

$\pi = 3.14159$. $\pi^2 = 9.8696$. $\frac{1}{\pi} = 0.3183$. $\sqrt{\pi} = 1.7724$.

Circumference of a circle (r = radius) $= 2\pi r$.
Area of a circle $= \pi r^2$.
Surface of a sphere $= 4\pi r^2$.
Volume of a sphere $= \frac{4}{3}\pi r^3$.
Volume of a right cylinder $= \pi r^2 h$.

Reductions

Meters × 3.2809 = feet.
Feet × 0.3048 = meters.
Cu. in. × 16.386 = ccm.
Cc. × 0.06103 = cu. in.
Oz. avoir. × 28.35 = grams.
Lbs. × 453.593 = grams.
Kg. × 2.2046 = lbs. avoir.
In. × 2.54 = cm.
Cm. × 0.3937 = in.

TABLE I

Tenacity of Solids

(Tenacity in dynes per sq. cm.)

	Multiply by $\overline{10}^{9}$
Aluminum	2.0
Brass (cast)	1.2
(hard drawn)	4.–9.
Copper (cast)	1.3
(hard drawn)	3.–5.
German silver	3.0
Glass	.3–.6
Iron (gray cast)	1. (?)
(wrought)	4.–7.
Lead (pressed)	0.2
Platinum	3.0
Steel (cast)	8.–10.
(tempered)	7.0
(wire)	10.–20.
Wood	.8–.1
Zinc	.5–1.5

TABLE II

Young's Modulus of Elasticity

Brass (wire)	$1.0 \times \overline{10}^{12}$ dynes.
Copper (wire)	$1.2 \times \overline{10}^{12}$ dynes.
Iron (wrought)	$1.9 \times \overline{10}^{12}$ dynes.
Steel (wire)	$2.8 \times \overline{10}^{12}$ dynes.

TABLE III

Local Values of Acceleration due to Gravity

Locality.	Latitude.	Elevation.	Value of G.
Boston, Mass.	42° 21′	22 meters.	980.382 cm.
Philadelphia, Pa.	39 57	16 "	980.182 "
Washington, D. C.	38 53	10 "	980.100 "
Cleveland, Ohio	41 30	210 "	980.227 "
Cincinnati, Ohio	39 08	245 "	979.990 "
Chicago, Ill.	41 47	182 "	980.264 "
St. Louis, Mo.	38 38	154 "	979.987 "
Kansas City, Mo.	39 05	278 "	979.976 "
Denver, Col.	39 40	1,638 "	979.595 "
San Francisco, Cal.	37 47	114 "	979.951 "
Greenwich, England	51 29	47 "	981.170 "
Paris	48 50	72 "	980.960 "
Berlin	52 30	35 "	981.240 "
Vienna	48 12	150 "	980.852 "
Rome	41 53	53 "	980.312 "
Hammerfest	70 40	Sea-level.	982.580 "
Latitude 45°	45 00	"	980.61 "
Equator	00 00	"	978.10 "
Pole	90 00	"	983.11 "

TABLE IV

Specific Gravity of Solids

Aluminum	2.7	Gold (pure)	19.3
Brass (cast)	8.3	Human body	0.89–1.07
(hard-drawn)	8.5	Ice	0.92
Bronze	8.8	Iron (gray cast)	7.5
Coal (anthracite)	1.26–1.8	(wrought)	7.8
(bituminous)	1.27–1.4	Lead (pressed)	11.3
Copper (cast)	8.8	Platinum	21.5
(hard-drawn)	8.8	Silver (pure)	10.5
Cork	0.2	Steel (cast)	7.8
Diamond	3.5	(wire)	7.9
German silver	8.5	Tin	7.3
Glass (crown)	2.5	Wood	.4–.7
(flint)	3.5	Zinc	7.0
Gold (75 per cent)	15.0		

TABLE V

Specific Gravity of Liquids

Acetic acid	at 15° C	1.05
Alcohol (ethyl)	“ “	0.79
(amyl)	“ “	0.81
Benzine	“ “	0.89
Carbon disulphide	“ “	1.27
Ether	“ “	0.72
Glycerine	“ “	1.26
Hydrochloric acid	“ “	1.22
Kerosene	“ “	0.78–0.84
Mercury	“ 0°	13.596
Milk (cow's)	“ “	1.032
Nitric acid	“ 15°	1.52
Linseed-oil	“ “	0.94
Turpentine	“ “	0.87
Sulphuric acid	“ “	1.84
Vinegar	“ “	1.026
Water (sea)	“ 0°	1.027
“ (pure)	“ “	0.999

TABLE VI

Specific Gravity of Gases

(Referred to air at 0° C. and 760 mm.)

Air	1.000
Ammonia	0.5967
Carbon dioxide	1.524
Coal-gas	0.34–0.65
Hydrogen	0.0692
Nitrogen	0.970
Oxygen	1.105

TABLE VII

Density of Water at Different Temperatures

Temperature.	Density.	Temperature.	Density.
0° C.	0.999884	23° C.	0.997610
1	0.999941	24	0.997380
2	0.999982	25	0.997125
3	1.000004	26	0.996879
4	1.000013	27	0.996609
5	1.000003	28	0.996344
6	0.999983	29	0.996052
7	0.999946	30	0.995778
8	0.999899	35	0.99419
9	0.999837	40	0.99236
10	0.999760	45	0.99038
11	0.999668	50	0.98821
12	0.999562	55	0.98583
13	0.999443	60	0.98339
14	0.999312	65	0.98075
15	0.999173	70	0.97795
16	0.999015	75	0.97499
17	0.998854	80	0.97195
18	0.998667	85	0.96880
19	0.998473	90	0.96557
20	0.998272	95	0.96202
21	0.998050	100	0.95866
22	0.997839		

TABLE VIII

Coefficients of Expansion of Solids *

	Linear.	Cubical.
Aluminum	0.000023	0.000071
Brass	0.0000185	0.000055
Copper	0.000017	0.000052
German silver	0.000018	0.000056
Glass	0.0000087	0.000025
Gold	0.000014	0.000047
Iron	0.000012	0.000038
Lead	0.000029	0.000086
Platinum	0.000009	0.000027
Silver	0.000019	0.000060
Steel	0.000010	0.000032
Tin	0.000023	0.000065
Wood (with the grain)	0.000005	0.000016
Zinc	0.000025	0.000088

* Average values of the range of 0° to 100° C.

TABLE IX

Coefficients of Expansion of Liquids

Alcohol (0° to 50° C.)	0.0011
Benzine	0.0014
Carbon disulphide	0.0015
Glycerine	0.00053
Mercury (0° to 100° C.)	0.00018
Turpentine	0.0011
Water (at 15° C.)	0.00012

TABLE X

Coefficients of Expansion of Gases

(Between 0° and 100° C.)

All true gases	about 0.00366

TABLE XI

Melting Points

	° C.
Aluminum	850
Antimony	430
Bismuth	260–270
Brass	1015–1030
Copper	1050–1100
Fusible metal (Rose's) *	94
Glass	1100
Iron	1200–1800
Lead	325
Mercury	39
Paraffine	54
Platinum	1750–1780
Tin	225–230
Zinc	400–420

---

* Four parts of bismuth, one of lead, and one of tin.

Table XII

Boiling-points

(Pressure 760 mm.)

Alcohol	77.9° C.
Ether	34.9
Mercury	350.0
Water (distilled)	100.0
(saturated with common salt)	103.0
(saturated with calcium chloride)	179.0

Table XIII

Specific Heats of Solids

Aluminum	0.2122° C.
Brass	0.0939
Copper	0.0950
Glass	0.198
Ice	0.504
Iron	0.1124
Lead	0.0314
Steel	0.118
Zinc	0.0935

Table XIV

Specific Heats of Liquids

Alcohol (at 0° C.)	0.5475
Ether (at 0° C.)	0.5468
Mercury (at 30° C.)	0.0333
Water	1.0000

Table XV

Latent Heats of Fusion

Copper (cast)	30.0	gram	calories.
Ice	79.25	“	“
Iron (gray cast)	23–25.0	“	“
(white cast)	33–34.0	“	“
Lead	5.35– 6.0	“	“
Zinc	28.0 +	“	“

TABLE XVI

Latent Heats of Vaporization

Alcohol	202.4	gram calories at	77.9° C.
Mercury	62.0	“ “ “	350.0° C.
Water	536.5	“ “ “	100.0° C.
Water	464.3	“ “ “	200.0° C.

TABLE XVII

*Size and Resistance of Pure Copper Wire, American Gauge (B. & S.)**

No.	DIAM. in mm.	RESISTANCE IN OHMS		No.	DIAM. in mm.	RESISTANCE IN OHMS	
		per 1000 feet.	per 1000 m.			per 1000 feet.	per 1000 m.
1	7.348	.129	.423	21	.723	13.323	43.699
2	6.544	.163	.534	22	.644	16.799	55.090
3	5.827	.205	.672	23	.573	21.185	69.486
4	5.189	.259	.849	24	.511	26.713	87.618
5	4.621	.326	1.059	25	.455	33.684	110.483
6	4.115	.411	1.348	26	.405	42.477	141.324
7	3.665	.519	1.702	27	.361	53.563	175.886
8	3.265	.654	2.145	28	.321	67.542	221.537
9	2.907	.824	2.702	29	.286	85.170	279.357
10	2.588	1.040	3.411	30	.255	107.391	352.242
11	2.305	1.311	4.300	31	.227	135.402	444.118
12	2.053	1.653	5.421	32	.202	170.765	540.100
13	1.828	2.084	6.835	33	.180	215.312	706.223
14	1.628	2.628	8.619	34	.160	271.583	890.822
15	1.450	3.314	10.962	35	.143	342.443	1133.213
16	1.291	4.179	13.706	36	.127	431.712	1416.015
17	1.150	5.269	17.282	37	.113	544.287	1785.261
18	1.024	6.645	21.780	38	.101	686.511	2250.756
19	.899	8.617	28.265	39	.090	865.046	2837.350
20	.812	10.566	34.656	40	.080	1091.865	3581.317

* At 24° C.

TABLE XVIII

Resistance of Wires *

(Chemically pure, 1 m. long, 1 mm. in diameter, at 0° C.)

		Relative to silver.
Copper (hard drawn)	0.02104 ohm	1.086
German silver	0.26950 "	13.92
Iron (annealed)	0.12510 "	6.46
Lead	0.25270 "	13.05
Platinum	0.11660 "	6.022
Silver	0.01937 "	1.000

TABLE XIX

For computing the Resistance of Wires

	Values of K at 20° C.
Aluminum	19.00
Copper	10.45
German silver	128.29
Iron	63.35
Lead	120.28
Mercury	58.24
Platinum	59.02
Silver	9.84

The resistance of a wire may be computed by means of the equation,

$$R = \frac{Kl}{d^2},$$

in which K is a constant depending upon the material and condition of the wire, l is the length of the wire in feet, and d is its diameter in thousandths of an inch.

---

* Jenkin and Ayrton.

TABLE XX

Temperature Coefficients of Resistance

Aluminum	0.00390
Carbon	0.0004 to 0.0005
Copper	0.00388
German silver	0.0004
Iron	0.00463
Iron-manganese-copper alloy *	Practically zero.
Mercury	0.00088
Platinum	0.00247
Platinoid †	0.00022

TABLE XXI

Approximate Electromotive Force of Primary Batteries

	Volts.
Daniell, amal. zinc, $H_2SO_4 + 4$ Aq., $CUSO_4$ concentrated sol.	1.079
Daniell, " " $H_2SO_4 + 12$ Aq., " " "	0.978
Leclanché, zinc in saturated solution of NH_4Cl	1.32
Potassium bichromate	2.00
Edison-Lalande	0.70

TABLE XXII

Velocity of Sound at 0° C.

Air	332	meters per sec.
Brass	3,318	" " "
Copper	3,556	" " "
Glass	4,526	" " "
Iron	5,127	" " "
Steel	5,237	" " "
Water (at 8°.1 C.)	1,435	" " "
Wood	3,300–4,700	" " "

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* Eighteen per cent ferro-manganese according to Nichols.

† German silver with one or two per cent of tungsten.

TABLE XXIII

Indices of Refraction

(For the mean D line.)

Air	1.000294
Alcohol	1.361
Carbon disulphide	1.626
Crown glass	1.515
Diamond	2.47–2.75
Flint glass	1.54–1.71
Water	1.332

TABLE XXIV

Trigonometrical Functions *

Angle.	Sine.	Difference.	Tangent.	Difference.	Cotangent.	Difference.	Cosine.	Difference.	
0°	0.000	17	0.000	17	∞		1.000	0	90°
1	0.017	18	0.017	18	57.29		1.000	0	89
2	0.035	17	0.035	17	28.64		0.999	0	88
3	0.052	18	0.052	18	19.08		0.999	1	87
4	0.070	17	0.070	17	14.30		0.998	2	86
5	0.087	18	0.087	18	11.43		0.996	1	85
6	0.105	17	0.105	18	9.514		0.995	2	84
7	0.122	17	0.123	18	8.144		0.993	3	83
8	0.139	17	0.141	17	7.115	811	0.990	2	82
9	0.156	18	0.158	18	6.314	643	0.988	3	81
10°	0.174	17	0.176	18	5.671	526	0.985	3	80°
11	0.191	17	0.194	19	5.145	440	0.982	4	79
12	0.208	17	0.213	18	4.705	374	0.978	4	78
13	0.225	17	0.231	18	4.331	320	0.974	4	77
14	0.242	17	0.249	16	4.011	279	0.970	4	76
15	0.259	17	0.268	19	3.732	245	0.966	5	75
16	0.276	16	0.287	19	3.487	216	0.961	5	74
17	0.292	17	0.306	19	3.271	193	0.956	5	73
18	0.309	17	0.325	19	3.078	174	0.951	5	72
19	0.326	16	0.344	20	2.904	157	0.946	6	71
	Cosine.		Cotangent.		Tangent.		Sine.		Angle.

---

* To find the functions of angles which exceed 45°, read upward in the column marked at the bottom with the desired function. The

TABLE XXIV—(*Continued*)

Angle.	Sine.	Difference.	Tangent.	Difference.	Cotangent.	Difference.	Cosine.	Difference.	
20°	0.342		0.364		2.747		0.940		70°
21	0.358	16	0.384	20	2.605	142	0.934	6	69
22	0.375	17	0.404	20	2.475	130	0.927	7	68
23	0.391	16	0.424	20	2.356	119	0.921	6	67
24	0.407	16	0.445	21	2.246	110	0.914	7	66
25	0.423	16	0.466	21	2.145	101	0.906	8	65
26	0.438	15	0.488	22	2.050	95	0.899	7	64
27	0.454	16	0.510	22	1.963	87	0.891	8	63
28	0.469	15	0.532	22	1.881	82	0.883	8	62
29	0.485	16	0.554	22	1.804	77	0.875	8	61
30°	0.500	15	0.577	23	1.732	73	0.866	9	60°
31	0.515	15	0.601	24	1.664	67	0.857	9	59
32	0.530	15	0.625	24	1.600	64	0.848	9	58
33	0.545	15	0.649	24	1.540	60	0.839	9	57
34	0.559	14	0.675	26	1.483	57	0.829	10	56
35	0.574	15	0.700	25	1.428	55	0.819	10	55
36	0.588	14	0.727	27	1.376	52	0.809	10	54
37	0.602	14	0.754	27	1.327	49	0.799	10	53
38	0.616	14	0.781	27	1.280	47	0.788	11	52
39	0.629	13	0.810	29	1.235	45	0.777	11	51
40°	0.643	14	0.839	29	1.192	43	0.766	11	50°
41	0.656	13	0.869	30	1.150	42	0.755	11	49
42	0.669	13	0.900	31	1.111	39	0.743	12	48
43	0.682	13	0.933	33	1.072	39	0.731	12	47
44	0.695	13	0.966	32	1.036	36	0.719	12	46
45°	0.707	12	1.000	35	1.000	36	0.707	12	45°
	Cosine.		Cotangent.		Tangent.		Sine.		Angle.

function of the fractional part of an angle may be found by multiplying the difference between the function of the integral portion of the angle and that of the next one in order by the decimal part of the angle. The differences just referred to, for all angles up to 90°, are given in this table.

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